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INTRODUCTION TO PROBABILITY DISTRIBUTIONS

1] BINOMIAL DISTRIBUTIONS
2] POISSON DISTRIBUTIONS
3] NORMAL DISTRIBUTIONS

Random variable & use of expected value in decision making

Random Variables

A **random variable** represents a numerical value associated with each outcome of a probability distribution.

The function of assigning a real number to every sample point in the sample space is called as random variable

E.G.:

a) Tossing a 2 coin, outcome is 4
(HH, HT, TH, TT)

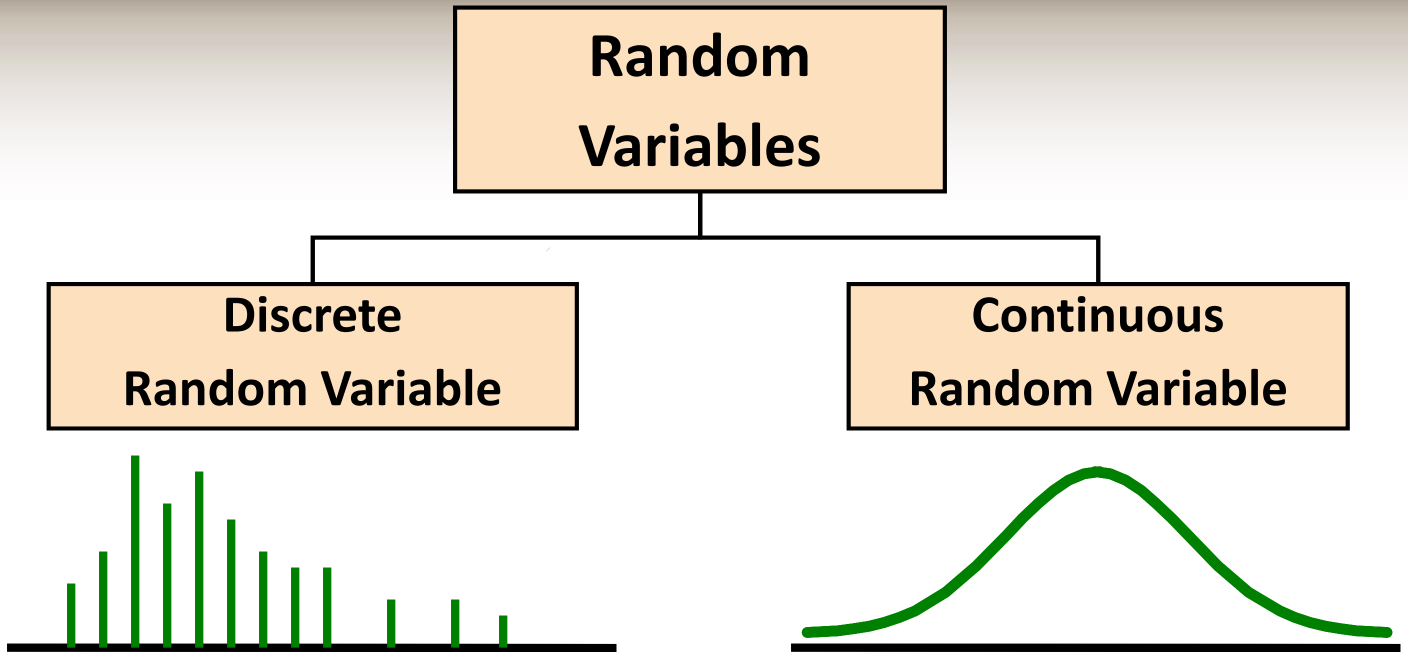
b)Throw a die twice; Count the number of times the number 6 appears (0, 1 or 2 times)

The number of Heads or number of dice is Denoted by “X”. It takes the values 0,1, & 2., Here “X” is called as random variable.

Expected value of a random variable

- Expected value is an extremely useful concept for good decision-making!
- Expected value is just the average or mean (μ) of random variable *x*.
- It's sometimes called a "weighted average" because more frequent values of X are weighted more highly in the average.

TYPES


Random Variables
Discrete Random Variable
Continuous Random Variable

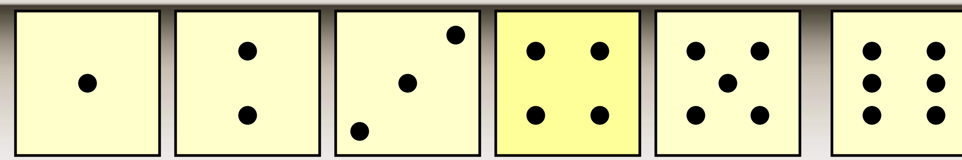
1] Discrete random variable

- A random variable is discrete if it has a finite or countable number of possible outcomes that can be listed.

- **Discrete** random variables produce outcomes that come from a counting process (e.g. number of classes you are taking).

- This probability function is also called as "**Probability Mass Function**" & its distribution is called as " **Discrete Probability Distribution**"

Examples:



- ➢Roll a die twice

  Let X be the number of times 4 occurs
  (then X could be 0, 1, or 2 times)

- ➢Toss a coin 5 times.

  Let X be the number of heads
  (then X = 0, 1, 2, 3, 4, or 5).

- ➢The number of students in a statistics class

  Let X be the number of students
  (then X = 10,15,20……)

<u>2] Continuous random variable</u>

- ✓ A random variable is continuous if it has an infinite or uncountable number or possible outcomes, represented by the intervals on a number line.

- ✓ **Continuous** random variables produce outcomes that come from a measurement (e.g. your annual salary, or your weight).

- ✓ This probability function is also called as "**Probability Density Function**" & its distribution is called as " **Continuous Probability Distribution**"

A continuous random variable is a variable that can assume any value on a an uncountable number of values.

- thickness of an item
- time required to complete a task
- temperature of a solution
- height, in inches

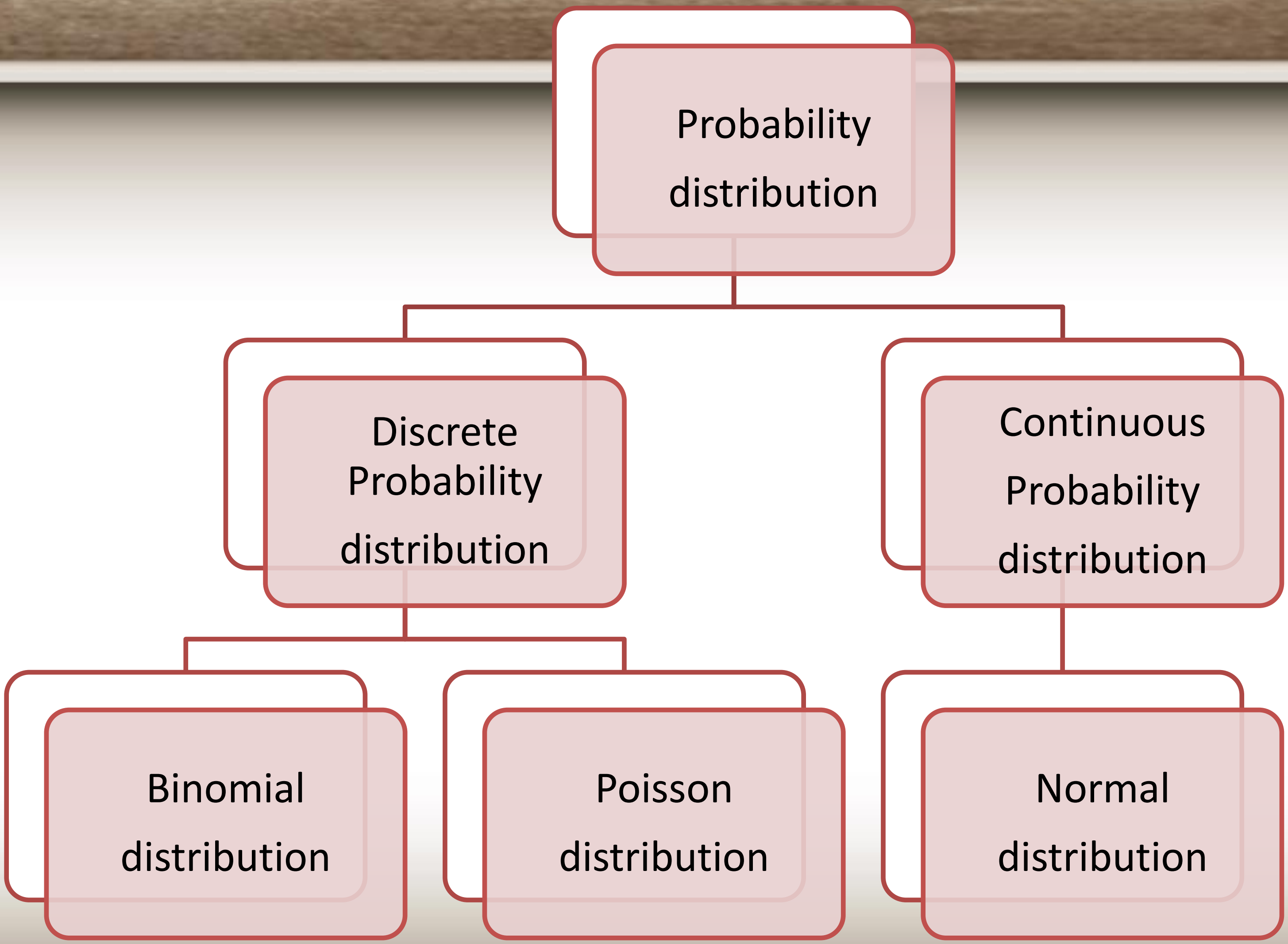
Example:

A distance of your car travels

The distance your car travels is a continuous random variable because it is a measurement that cannot be counted. (All measurements are continuous random variables.)

These can potentially take on any value depending only on the ability to precisely and accurately measure.

TYPES OF THEORETICAL PROBABILITY DISTRIBUTION



1]Binomial Distribution:-

It is a discrete probability distribution. It is also called as "Bernoulli Distribution".

It expresses the probability of one set of different alternatives I,e success [p] or failure [q]. The probability of occurrence of an event is "p" & its non occurrence is "q".

Binomial distribution as the prefix "Bi" shows 2 possibilities that is, the distribution in which the probability of an event is either to happen or not to happen i,e either success or failure & no other alternative results.

Examples

A] Toss of a coin

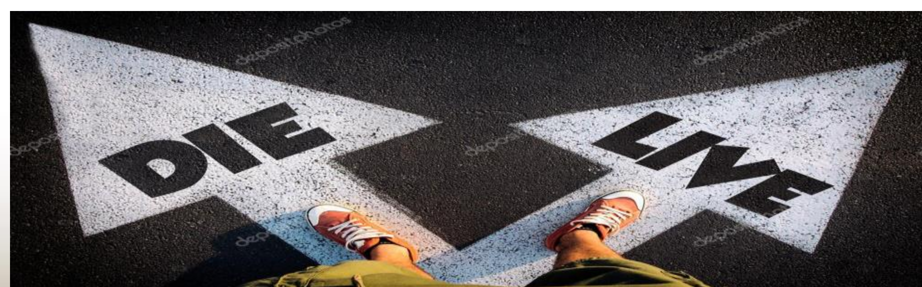
(Head or Tail)

B] Result of a student appearing in a test

(Pass or Fail)

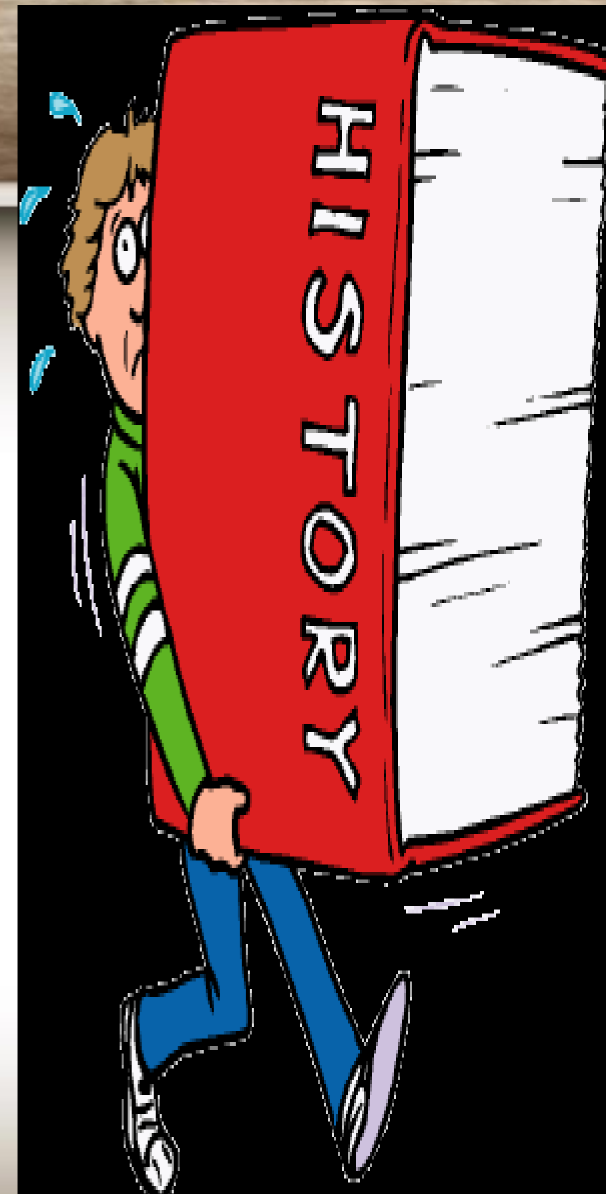
C]Survival of an organism in a region

(Live or Die)



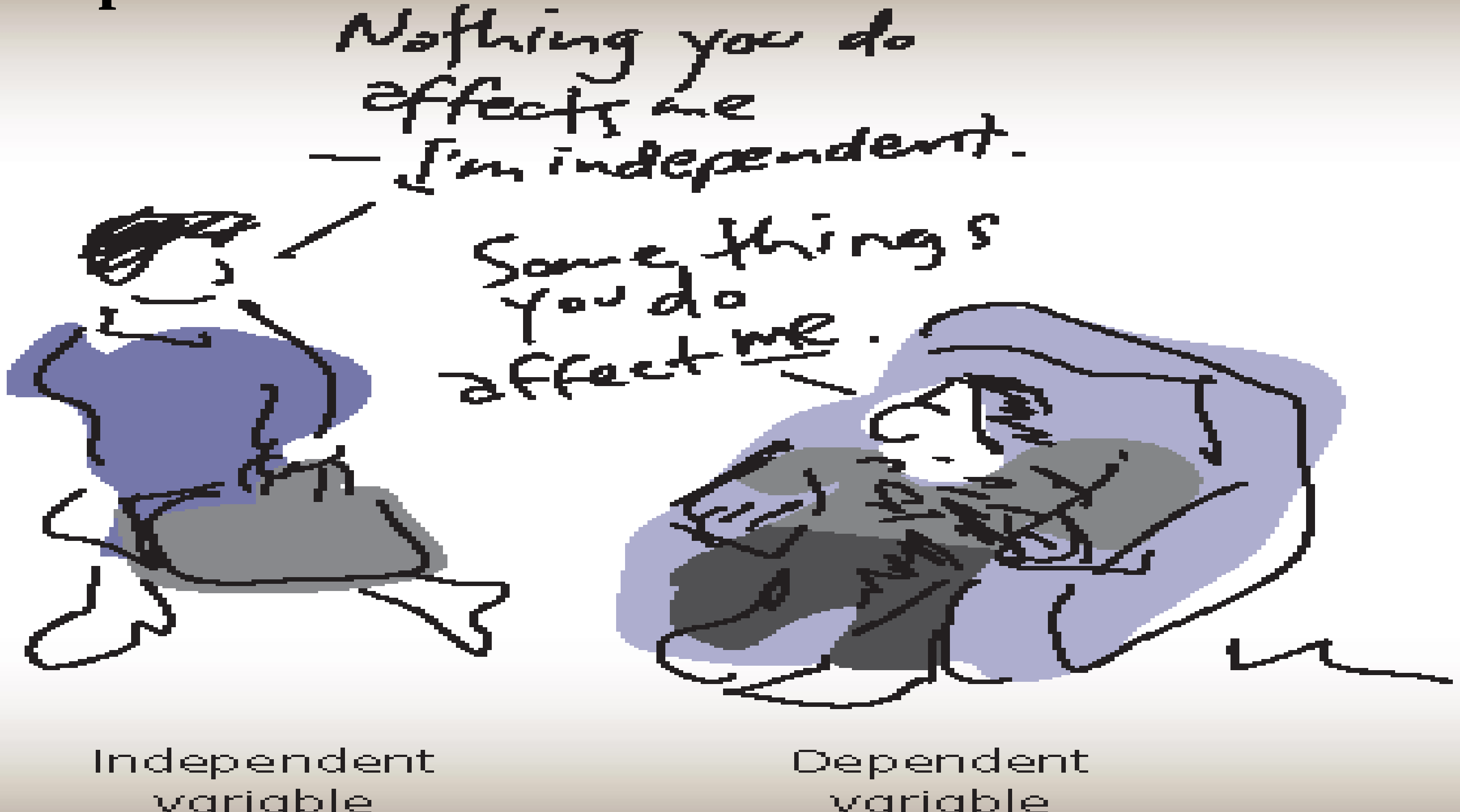
HISTORY OF BINOMIAL

- Swiss mathematician James Bernoulli [1654 – 1705], in a proof published in 1713, determined that the probability.
- After his death. Hence the name binomial distribution was propounded.
- The work of James was published in ARTS CONJECTANDI IN BASEL IN 1713.



When to use the binomial distribution

Independent variables



Features

- It is a discrete distribution which gives the theoretical probabilities.
- It depends on the 2 parameters p or q
- Mode of binomial distribution is equal to value of x which has largest frequency
- Shape & location of a binomial distribution changes as ‘p’ changes for a given ‘n’ or ‘n’ changes for a given ‘p’.
- Binomial co – efficient are given by the Pascal’s Triangle
- It should be independent variable.

Importance or Uses

- It is very useful in decision making in the business world. It is widely used in quality control.
- It is useful in describing various events of real life based on information.
- It is useful in study of various measures of central tendency, variation, moments, skewness & kurtosis
- It helps in comparing the symmetry or asymmetry of the observed data with the expected frequency distribution
- It reveals that the possibility of outcome of any trial does not change & is independent of the results of previous trails.

Assumptions or conditions

- The number of trails is finite , fixed & small
- In every trail, the number of possible outcomes is only 2 [success or failure]
- The trials are independent, i,e the outcome of one trail does not affect the other trial
- The probability of success is denoted by ‘p’ & is fixed ‘q’ is the probability of failure & is equal to 1 – p
- The number of success ‘X’ in ‘n’ trails of a binomial experiment is called a binomial random variable
- The trails or events must be repeated under identical conditions.

APPLICATIONS OF BINOMIAL DISTRIBUTION

- If a new drug is introduced to cure a disease then it either cure the disease or doesn't.
- If a person purchase a lottery ticket then he is either going to win it or not.
- The number of successful sale calls.
- The no of defective products in a production run.
- The house agent can provide suitable accommodation for 75% out of clients approach him.
- To find out the defective events from the samples chosen
- Number of arrows hitting a target at a certain number of times out of 'n' times given.

Fitting a Binomial distribution

STEPS:

- Find out the value of **p & q**
- Expand the binomial $(q + p)^n$.
- Find out the value of $N (p + q)^n$
- **S** and **F** (success and failure) denote two possible categories of all outcomes.
- $P(S) = p$ (p = probability of success)
- $P(F) = 1 – p = q$ (q = probability of failure)
- n =denotes the number of fixed trials.

Binomial Probability Formula.

$$Np_r = N^nc_r\ p^r\ q^{\ n-r}$$

N = no. of times an experiment is repeated

n= no. of outcomes

p= rate of success

q= rate of failure

r= expected frequency of success

A binomial is a polynomial with two terms such as *x* + *a*. Often we need to raise a binomial to a power. In this section we'll explore a way to do just that without lengthy multiplication.

$$(x+a)^0 = 1$$

$$(x+a)^1 = x + a$$

$$(x+a)^2 = x^2 + 2ax + a^2$$

$$(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$$

$$(x+a)^4 = x^4 + 4ax^3 + 6a^2x^2 + 4a^3x + a^4$$

$$(x+a)^5 = x^5 + \_\_ax^4 + \_\_a^2x^3 + \_\_a^3x^2 + \_\_a^4x + a^5$$

Can you see a pattern?

Can you make a guess what the next one would be?

We can easily see the pattern on the *x*'s and the *a*'s. But what about the coefficients? Make a guess and then as we go we'll see how you did.

Let's list all of the coefficients on the *x*'s and the *a*'s and look for a pattern.

$$(x+a)^5 = 1x^5 + 5ax^4 + 10a^2x^3 + 10a^3x^2 + 5a^4x + 1a^5$$

$$(x+a)^0 = 1$$

$$(x+a)^1 = 1x + 1a$$

$$(x+a)^2 = 1x^2 + 2ax + 1a^2$$

$$(x+a)^3 = 1x^3 + 3ax^2 + 3a^2x + 1a^3$$

$$(x+a)^4 = 1x^4 + 4ax^3 + 6a^2x^2 + 4a^3x + 1a^4$$

1 + 1

1 + 2 + 1

1 + 3 + 3 + 1

1 + 4 + 6 + 4 + 1

1 5 10 10 5 1

Can you guess the next row?

This is good for lower powers but could get very large. We will introduce some notation to help us and generalise the coefficients with a formula based on what was observed here.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

This is called Pascal's Triangle and would give us the coefficients for a binomial expansion of any power if we extended it far enough.

Pascal's Triangle
$(a+b)^n$

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

nCr	
$_5C_0$	1
$_5C_1$	5
$_5C_2$	10
$_5C_3$	10
$_5C_4$	5
$_5C_5$	1

10 ways to get to the 3rd position numbering each of the terms from 0 to 5.
this can also be calculated by using nCr button on your calculator $_5C_2=10$

Pascal's Triangle
$(a+b)^n$

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

nCr	n!÷(c!x(n-c)!)	
5C0	5!÷(0!x5!)	1
5C1	5!÷(1!x4!)	5
5C2	5!÷(2!x3!)	10
5C3	5!÷(3!x2!)	10
5C4	5!÷(4!x1!)	5
5C5	5!÷(5!x0!)	1

Binomial Probability Theorem

The binomial theorem is used to calculate the probability for the outcomes of **repeated independent and identical trials**. If p is the probability of success and q is the probability of failure ($q = 1 - p$), then the probability of x successes in n trials is:

$$P(x \text{ successes}) = {}_nC_x\, p^x q^{n-x}$$

1. Hockey cards, chosen at random from a set of 20, are given away inside cereal boxes. Stan needs one more card to complete his set, so he buys five boxes of cereal. What is the probability that he will complete his set?

$n = 5$ **(number of trials)**

$x = 1$ **(number of successes)**

$p = \frac{1}{20}$ **(probability of succes**

$q = \frac{19}{20}$ **(probability of failure)**

$$P(x \text{ successes}) = {}_nC_x\, p^x q^{n-x}$$

$$P(1 \text{ success}) = {}_5C_1 \times \left(\frac{1}{20}\right)^1 \times \left(\frac{19}{20}\right)^4$$

$= 0.2$

The probability of Stan completing his set is 20%.

2. Seven coins are tossed. What is the probability of four tails and three heads?

$n =$ 7 (number of trials)

$x =$ 4 (number of successes)

$p = \frac{1}{2}$ (probability of success)

$q = \frac{1}{2}$ (probability of failure)

$P(x \text{ successes}) = {}_nC_x\, p^x\, q^{n-x}$

$P(4 \text{ successes}) = {}_7C_4 \times (\frac{1}{2})^4 \times (\frac{1}{2})^3$

= 0.273

The probability of four heads and three tails is 27%.

3. A true-false test has 12 questions. Suppose you guess all 12. What is the probability of exactly seven correct answers?

$n =$ 12 (number of trials)

$x =$ 7 (number of successes)

$p = \frac{1}{2}$ (probability of success)

$q = \frac{1}{2}$ (probability of failure)

$P(x \text{ successes}) = {}_nC_x\, p^x\, q^{n-x}$

$P(7 \text{ successes}) = {}_{12}C_7 \times (\frac{1}{2})^7 \times (\frac{1}{2})^5$

= 0.193

The probability of seven correct answers is 19%.

Properties of Binomial disribution

1] there are 2 parameters p& q where n = no. of outcomes

2]Mean, μ = n*p

3]Std. Dev. s = $\sqrt{n * p * q}$

4] Variance, =n*p*q

5] skewness

6] kurtosis

7] if p = q = 0.5 the binomial distribution is said to be symmetrical

8] if p < 0.5 it is positively skewed, if p> 0.5 it is negatively skewed

Poisson Distribution

Poisson Distribution

It is a discrete probability distribution & is very widely used in statistical work. It is been called as "**Law of improbable events**". It is also known as probability distribution of rare events.

It may be expected in cases where the chance of any individual event being a success is small.

It is a technique of calculating the probability of an event. It is applied when the probability of non occurrence of an event is impossible to predict or find out.

A Binomial distribution can be approximated by a Poisson distribution if 'n' is large & 'p' is small.

EXAMPLE:

- ✓The number of scratches in a car's paint
- ✓The number of mosquito bites on a person
- ✓The number of computer crashes in a day
- ✓Number of accidents in a road

HISTORY OF POISSON DISTRIBUTION

- Discovered by Mathematician Simeon D Poisson in France in 1837 [1781- 1840].
- the first application was the description of the number of deaths by horse kicking in the Prussian army.
- The modeling distribution that takes his name was originally derived as an approximation to the binomial distribution.
- Applied in death of infants, the number of misprints in a book, the number of customers arriving, and the number of activations of a Geiger counter.

Why Poisson distribution is used

- When there is a large number of trials, but a small probability of success, binomial calculation becomes impractical.

- Some events are rather rare - they don't happen that often.

  For instance, car accidents are the exception rather than the rule. Still, over a period of time, we can say something about the nature of rare events.

Example:

Number of the death from horse kicks in the army in different years.

CONDITIONS OR ASSUMPTIONS

- Counting the number of times a success occur in an interval
- Probability of success the same for all intervals of equal size
- Number of successes in interval independent of number of successes in other intervals
- Probability of success is proportional to the size of the interval
- Intervals do not overlap.
- No. of trials is indefinitely large $n \to \infty$
- Probability of success p for each trialis very small $p \to 0$
- Mean is a finite number given by $np = \lambda$

APPLICATIONS OF POISSON DISTRIBUTION

- It is used in quality control statistics to count the numbers of defects of any item
- In biology to count the number of bacteria
- In physics to count the number of particles emitted from a radio active substance
- In insurance problems to count the number of casualities
- In waiting time problems to count the number of incoming telephone number of incoming telephone calls or incoming customers
- The number of typographical errors per page in typed material

Features

- It is a discrete distribution
- It is applied in a situation where value of p is very small & q is very large
- It is a skewed distribution
- Main parameter of this distribution is mean = np
- It depends mainly on the value of the mean & the values are constant
- Its arithmatic mean in relative distribution is p & absolute distribution is np

Formula

$$P(r) = \frac{e^{-\mu}\mu^{r}}{r!}$$

$X \sim P(\lambda)$

$P(r)$ = probability of r "successes" given m

r : 0,1,2,3…. Expected number of times

m : mean - expected (average) number of "successes"

e: 2.71828 constant (base of natural logs)

λ [Pois] is the shape parameter which indicates the average number of events in the given time interval.

Properties of Poisson distribution

Mean $$\mu = \lambda$$

- Variance $$\sigma^2 = \lambda$$
- Standard Deviation $$\sigma = \sqrt{\lambda}$$
- Skewness $$\beta_1 = 1/\ m$$
- Kurtosis $$\beta_2 = 3 + 1/\ m$$
- $\mu_1 = 0$, $\mu_2 = m$, $\mu_3 = m$ & $\mu_4 = m + 3m^2$

(VALUE OF e^{-m}); $(0 < m < 1)$

m	0	1	2	3	4	5	6	7	8	9
0.0	1.0000	.9900	.9802	.9704	.9608	.9512	.9418	.9324	.9231	.9139
0.1	0.9048	.8958	.8860	.8781	.8694	.8607	.8521	.8437	.8353	.8270
0.2	0.8178	.8106	.8025	.7945	.7866	.7788	.7711	.7634	.7558	.7483
0.3	0.7408	.7334	.7261	.7189	.7118	.7047	.6977	.6907	.6839	.6771
0.4	0.6703	.6636	.6570	.6505	.6440	.6376	.6313	.6250	.6188	.6126
0.5	0.6065	.6005	.5945	.5886	.5827	.5770	.5712	.5655	.5599	.5543
0.6	0.5488	.5434	.5379	.5326	.5278	.5220	.5160	.5117	.5066	.5016
0.7	0.4966	.4916	.4868	.4810	.4771	.4724	.4670	.4630	.4584	.4538
0.8	0.4493	.4449	.4404	.4360	.4317	.4274	.4232	.4190	.4148	.4107
0.9	0.4066	.4025	.3985	.3946	.3906	.3867	.3829	.3791	.3753	.3716

VALUE OF e^{-m}; (m = 1, 2, 3, ..., 10)

m	1	2	3	4	5	6	7	8	9	10
e^{-m}	.36788	.13534	.04979	.01832	.006738	.002479	.000912	.000335	.000123	.000045

Note : To obtain value of e^{-m} for other values of m, use the laws of exponents.

Example : $e^{-2.35} = (e^{-2.00})(e^{-0.35}) = (0.13534)(0.7047) = 0.095374$

Table -VI Poisson Distribution - Exponents

λ	$e^{-\lambda}$	λ	$e^{-\lambda}$	λ	$e^{-\lambda}$
0.00	1.0000	2.1	0.1225	5.2	0.0055
0.01	0.9900	2.2	0.1108	5.4	0.0045
0.02	0.9802	2.3	0.1003	5.6	0.0037
0.03	0.9704	2.4	0.0907	5.8	0.0030
0.04	0.9608	2.5	0.0821	6.0	0.0025
0.05	0.9512	2.6	0.0743	6.2	0.0020
0.06	0.9418	2.7	0.0672	6.4	0.0017
0.07	0.9324	2.8	0.0608	6.6	0.0014
0.08	0.9231	2.9	0.0550	6.8	0.0011
0.09	0.9139	3.0	0.0498	7.0	0.00091
0.1	0.9048	3.1	0.0450	7.2	0.00075
0.2	0.8187	3.2	0.0408	7.4	0.00061
0.3	0.7408	3.3	0.0369	7.6	0.00050
0.4	0.6703	3.4	0.0334	7.8	0.00041
0.5	0.6065	3.5	0.0302	8.0	0.00034
0.6	0.5488	3.6	0.0273	8.2	0.00027
0.7	0.4966	3.7	0.0247	8.4	0.00023
0.8	0.4493	3.8	0.0224	8.6	0.00018
0.9	0.4066	3.9	0.0202	8.8	0.00015
1.0	0.3079	4.0	0.0183	9.0	0.00012
1.1	0.3329	4.1	0.0166	9.2	0.000101
1.2	0.3012	4.2	0.0150	9.4	0.000083
1.3	0.2725	4.3	0.0136	9.6	0.000068
1.4	0.2466	4.4	0.0123	9.8	0.000055
1.5	0.2231	4.5	0.0119	10.0	0.000045
1.6	0.2019	4.6	0.0101	10.2	0.000017
1.7	0.1827	4.7	0.0091	10.4	0.000006
1.8	0.1653	4.8	0.0082		
1.9	0.1496	4.9	0.0074		
2.0	0.1353	5.0	0.0067		

Help: $e^{-7.31} = e^{-7} \times e^{-0.3} \times e^{-0.01} = 0.00091 \times 0.7408 \times 0.9900 = 0.0006674$

CONTINUOUS PROBABILITY DISTRIBUTIONS

Normal Distribution

Normal Distribution

The normal distribution is a descriptive model that describes real world situations. It is defined as a continuous frequency distribution of infinite range.

This is the most important probability distribution in statistics & important tool in analysis of epidemiological data & management science.

It is mainly used to study the behaviour of continuous random variable like weight, height etc.. The normal distribution is an approximation to binomial distribution.

When the number of trials or experiments become large the occurrence of an event p & non occurrence of the event q tends to be equal.

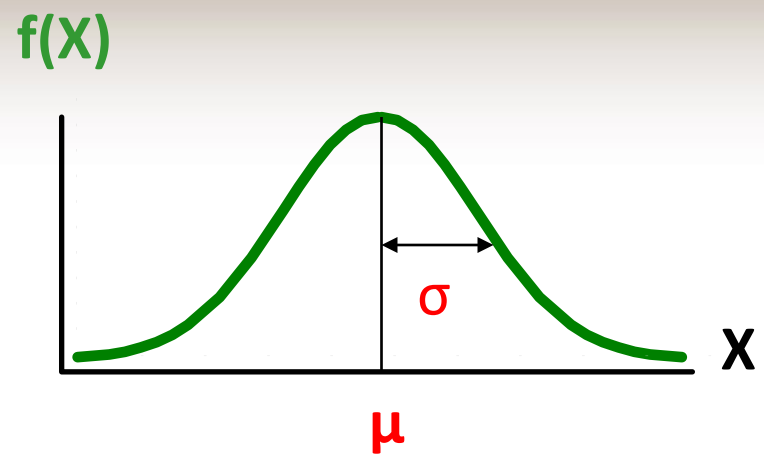
In such a situation the approximation of the events should be taken into consideration. The limiting frequency curve obtained as n becomes large is called normal distribution curve.

HISTROY

- 1ST discovered by Abrham De Moivre in 1733 { 1667- 1754] an English mathematician to solve the problems in games of chances.
- Rediscovered by Carl Friedrich Gauss an German mathematician in 1809 [1777 – 1855]. Who 1st developed a 2 parameter exponential function.He called as “ Prince of Mathematician”.
- And also La place an French mathematician in 1812 [1749 – 1827]
- Both rediscovered… it was applied in natural & social sciences.
- The normal distribution also as “Gaussian distribution” [Gaussian Law of Error] & “Gaussian Curve”

The Graph of Normal Distribution

- **‘Bell Shaped’**
- **Symmetrical**
- **Mean, Median and Mode are Equal**
- **Location is determined by the mean, μ**
- **Spread is determined by the standard deviation, σ**
- Classified by 2 parameters: Mean (μ) and standard deviation (σ).
- **The random variable has an infinite theoretical range:** $+\infty$ **to** $-\infty$



= Mean
= Median
= Mode

Characteristics of Normal Distribution

- It links frequency distribution to probability distribution.
- Has a Bell Shape Curve and is Symmetric.

- It is Symmetric around the mean:
  Two halves of the curve are the same (mirror images).
  Hence Mean = Median

- The total area under the curve is 1 (or 100%)

- Normal Distribution has the same shape as Standard Normal Distribution.
- In a Standard Normal Distribution:
  The mean (μ) = 0 and Standard deviation (σ) =1

The Standardized Normal

Any normal distribution (with any mean and standard deviation combination) can be transformed into the standardized normal distribution (Z)

Need to transform X units into Z units

The standardized normal distribution (Z) has a **mean of 0** and a **standard deviation of 1**

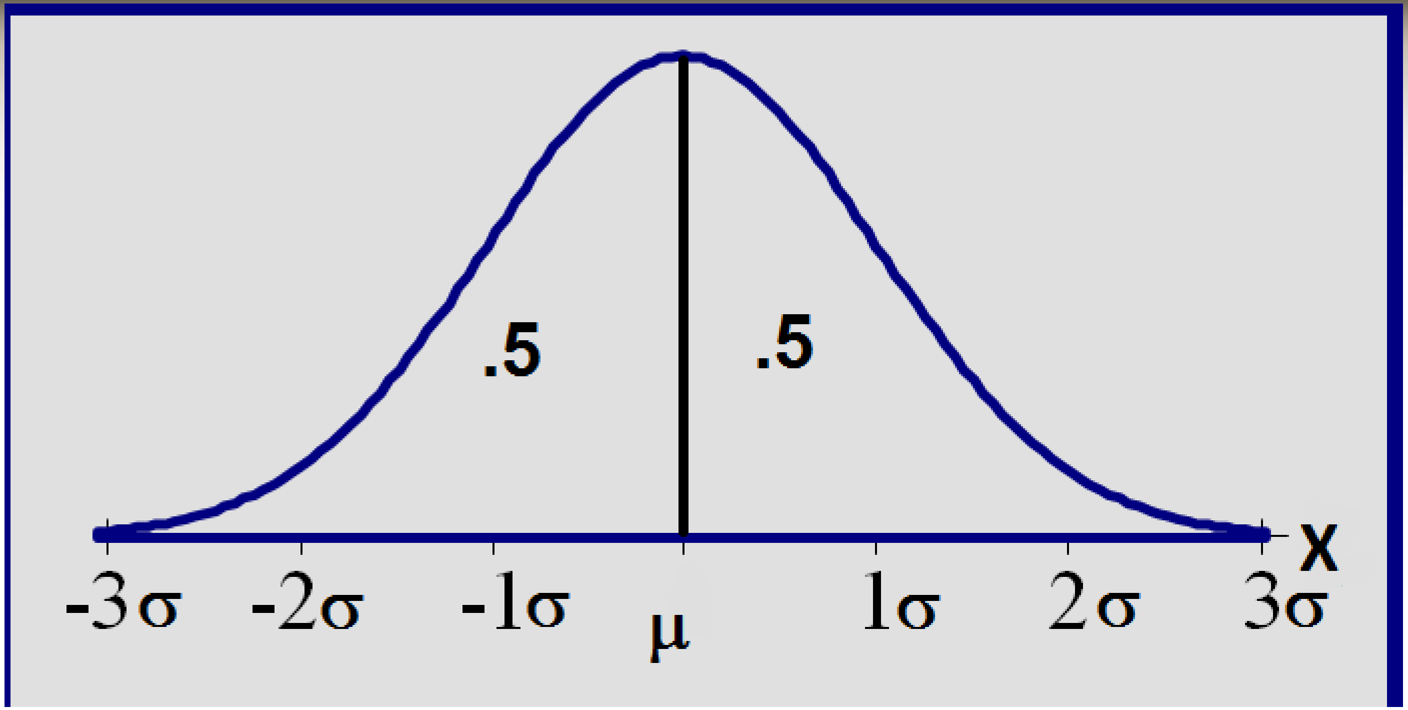
Translation to the Standardized Normal Distribution

Translate from X to the standardized normal (the "Z" distribution) by subtracting the mean of X and dividing by its standard deviation:

$$Z = \frac{X - \mu}{\sigma}$$

The Z distribution always has mean = 0 and standard deviation = 1

Total area =1; symmetric around μ



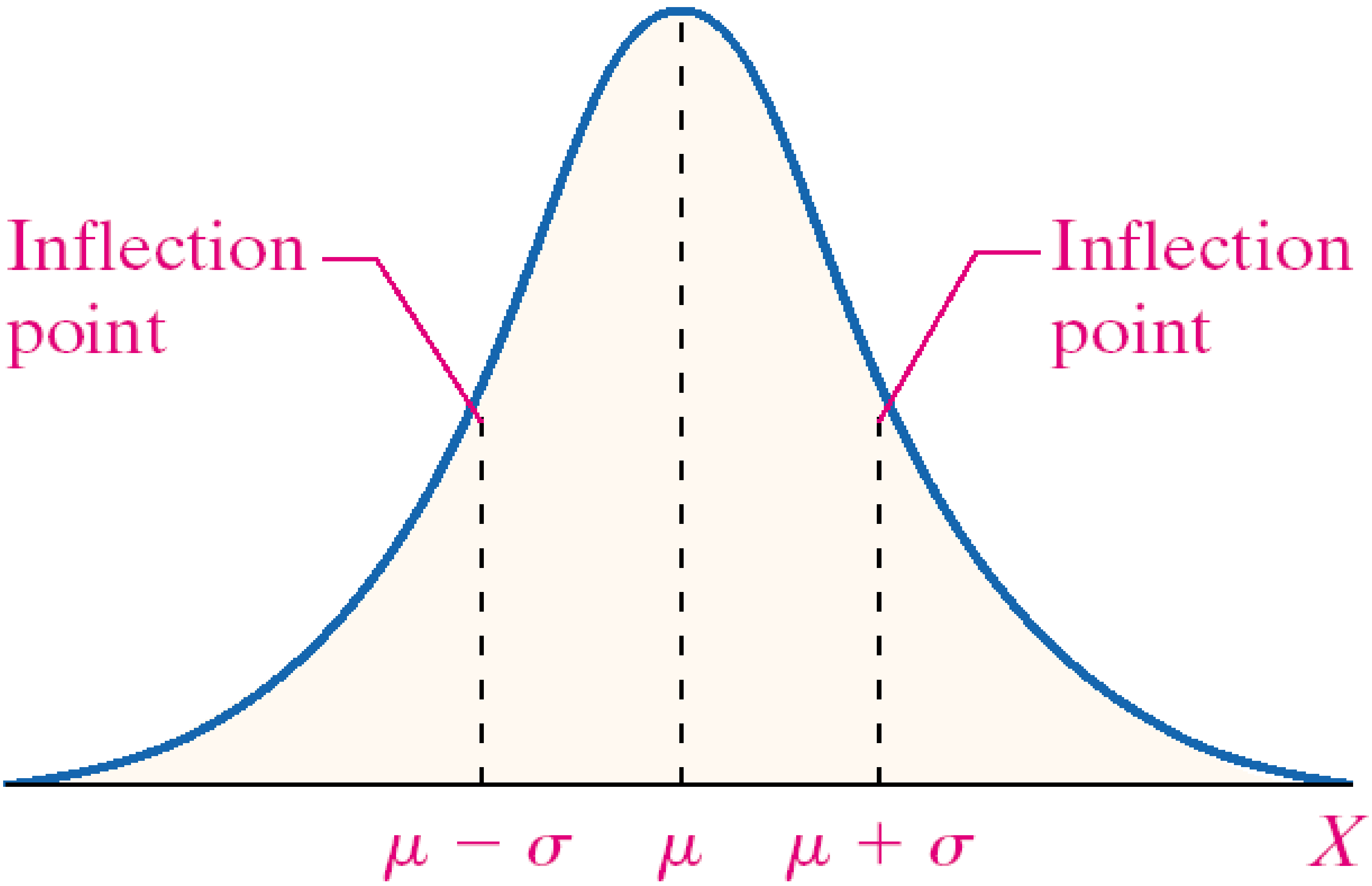
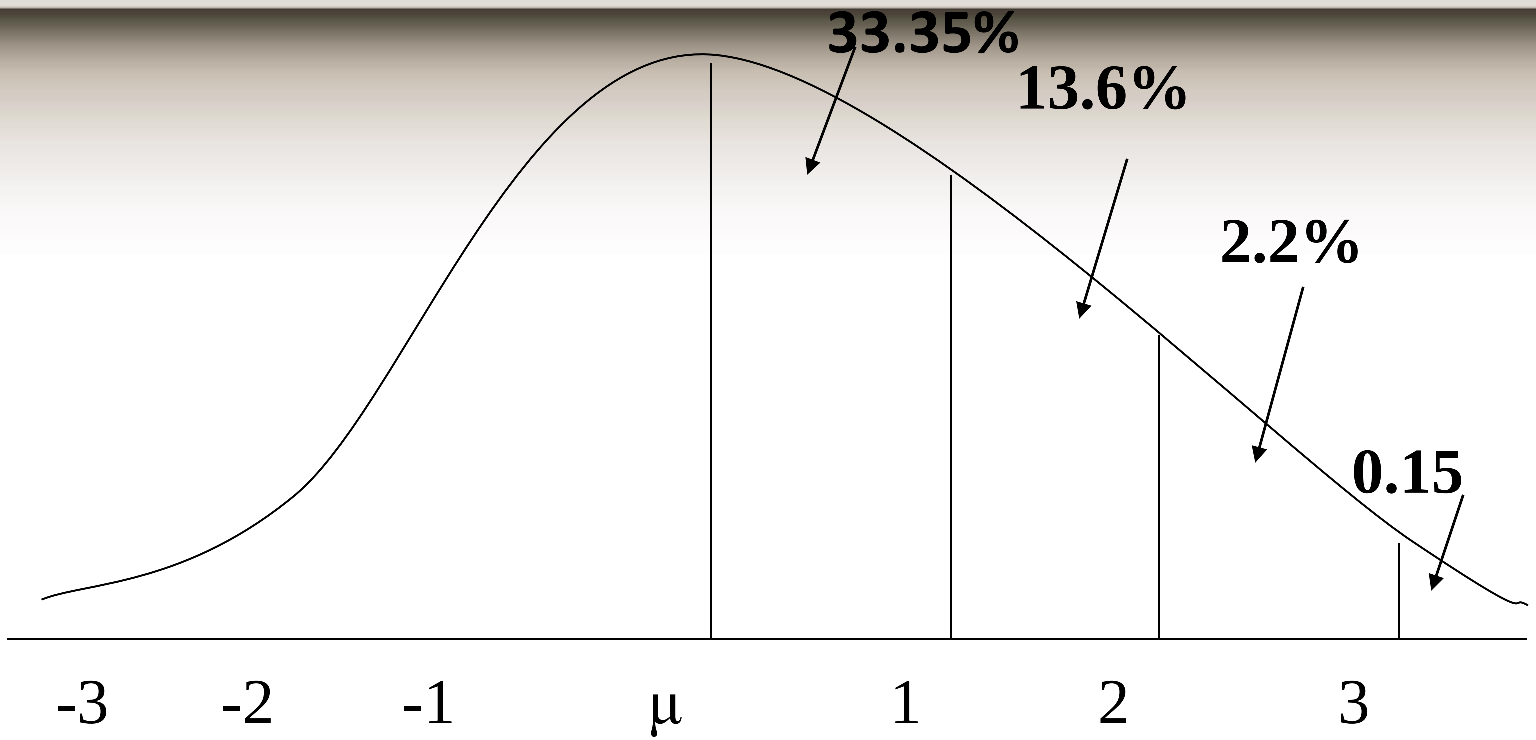

Inflection point
Inflection point
$\mu - \sigma$
μ
$\mu + \sigma$
X

Diagram of Normal Distribution Curve (z distribution)



33.35 + 13.6 + 2.2 + 0.15 % = 50%

Skewness

- **Positive Skewness: Mean ≥ Median**

- **Negative Skewness: Median ≥ Mean**

- **Pearson's Coefficient of Skewness[3]:**

$$= \frac{3\,(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$

Properties of the Normal Density Curve

1. It is symmetric about its mean, μ.

2. The highest point occurs at $x = \mu$

3. It has inflection points at $\mu - \sigma$ and $\mu + \sigma$.

4. The area under the curve is one.

5. The area under the curve to the right of μ equals the area under the curve to the left of μ equals ½.

6. As x increases without bound, the graph approaches, but never equals, zero. As x decreases without bound the graph approaches, but never equals, zero.

Properties of the Normal Density Curve

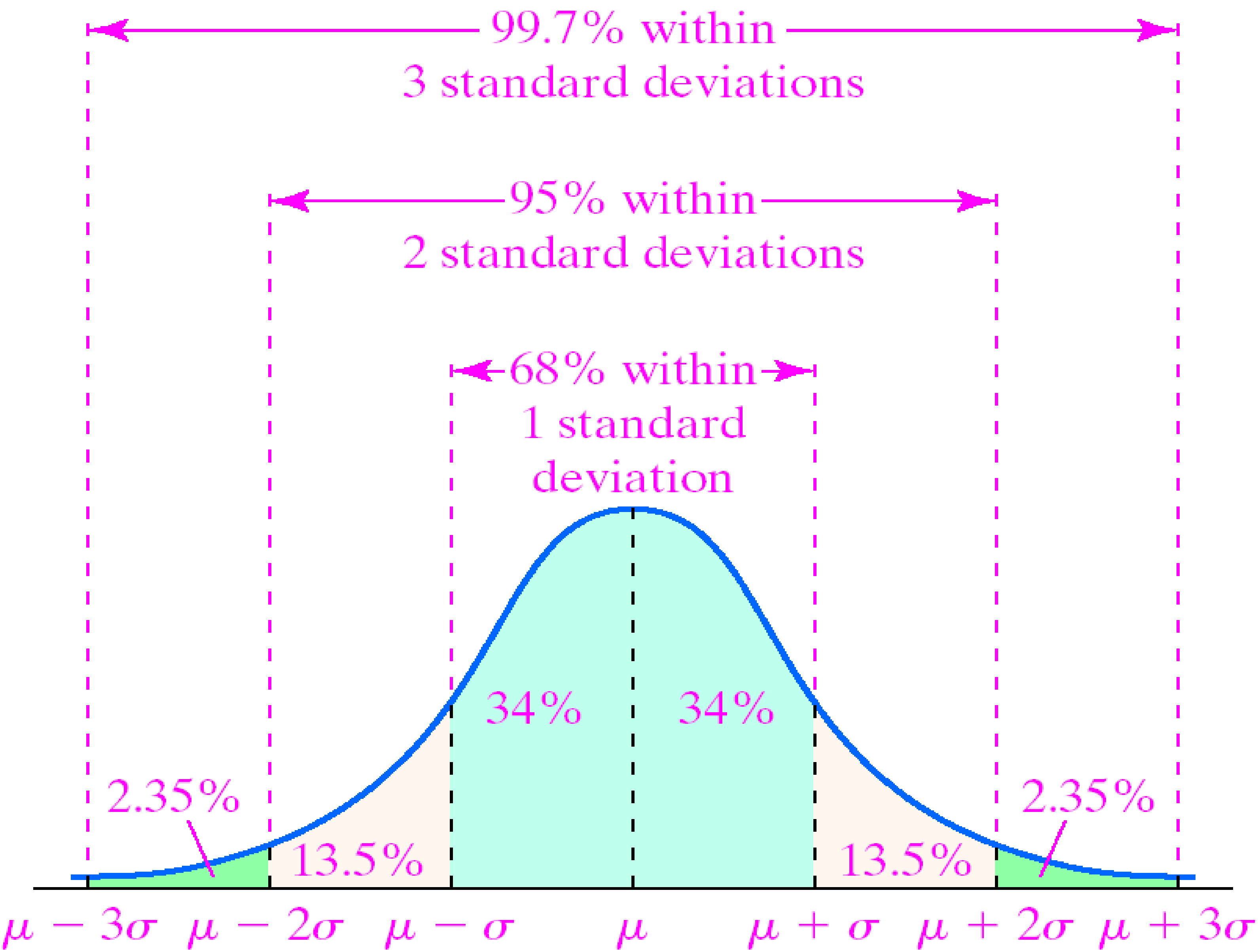
7. The Empirical Rule:

About 68% of the area under the graph is within one standard deviation of the mean.

About 95% of the area under the graph is within two standard deviations of the mean.

About 99.7% of the area under the graph is within three standard deviations of the mean.

Normal Distribution



The beauty of the normal curve:

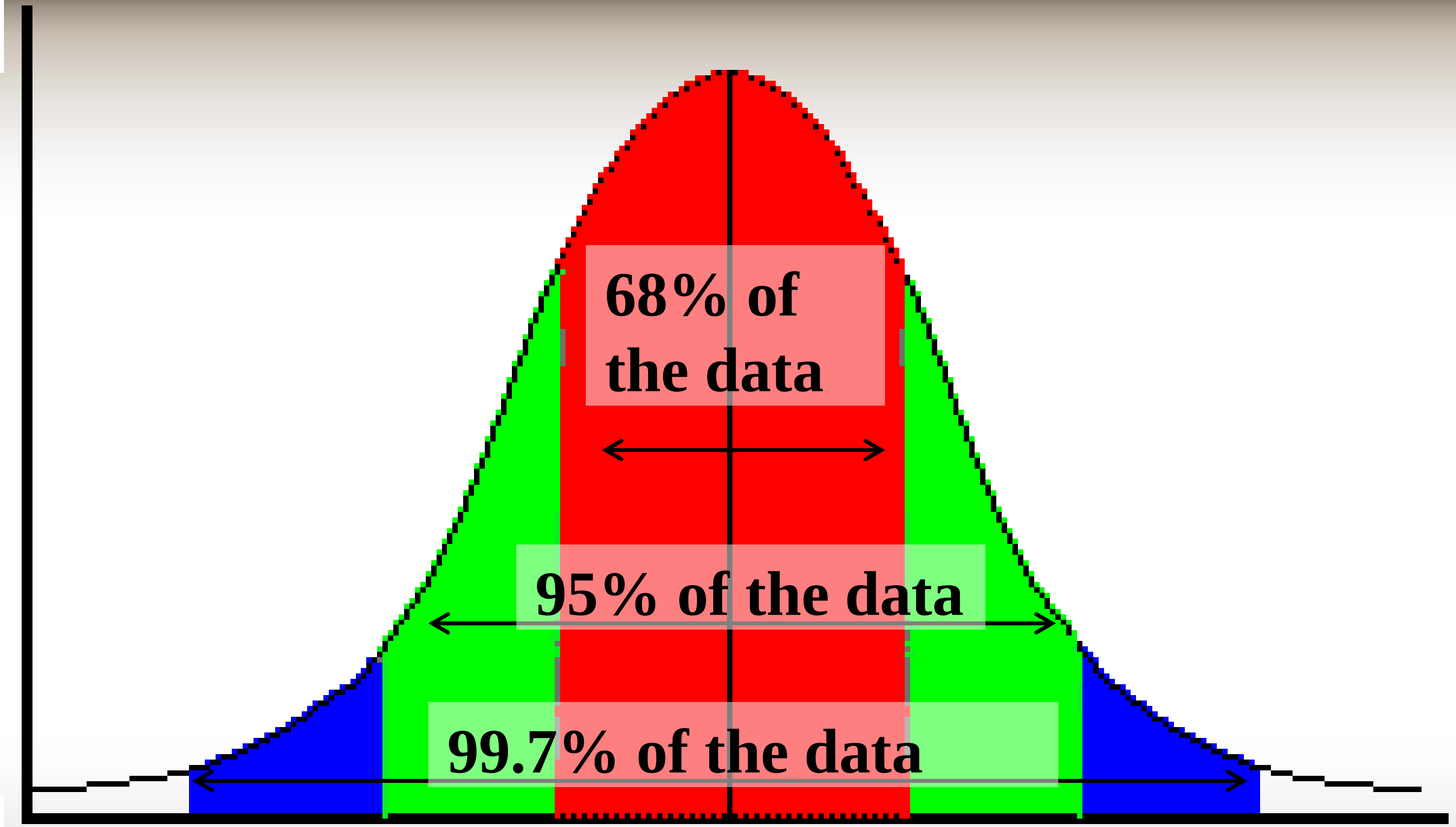
No matter what μ and σ are, the area between μ-σ and μ+σ is about 68%;

the area between μ-2σ and μ+2σ is about 95%; and

the area between μ-3σ and μ+3σ is about 99.7%.

Almost all values fall within 3 standard deviations.

68-95-99.7 Rule



Application/Uses of Normal Distribution

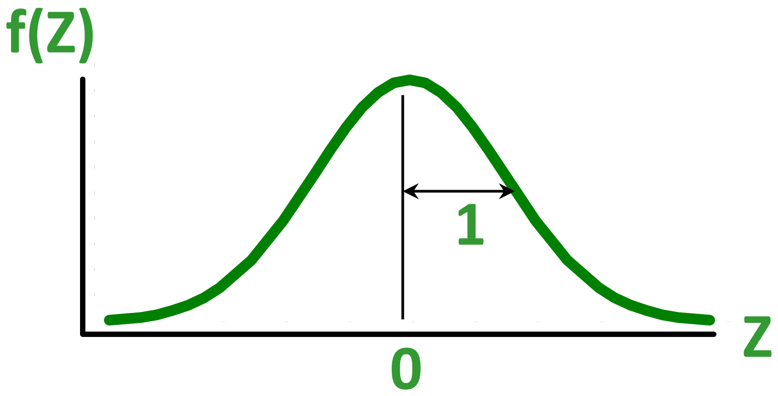
- **It's application goes beyond describing distributions**
- **It is used by researchers and modelers.**
- **The major use of normal distribution is the role it plays in statistical inference.**
- **The z score along with the t –score, chi-square and F-statistics is important in hypothesis testing.**
- **It helps managers/management make decisions.**

The Standardized Normal Distribution

Also known as the "Z" distribution

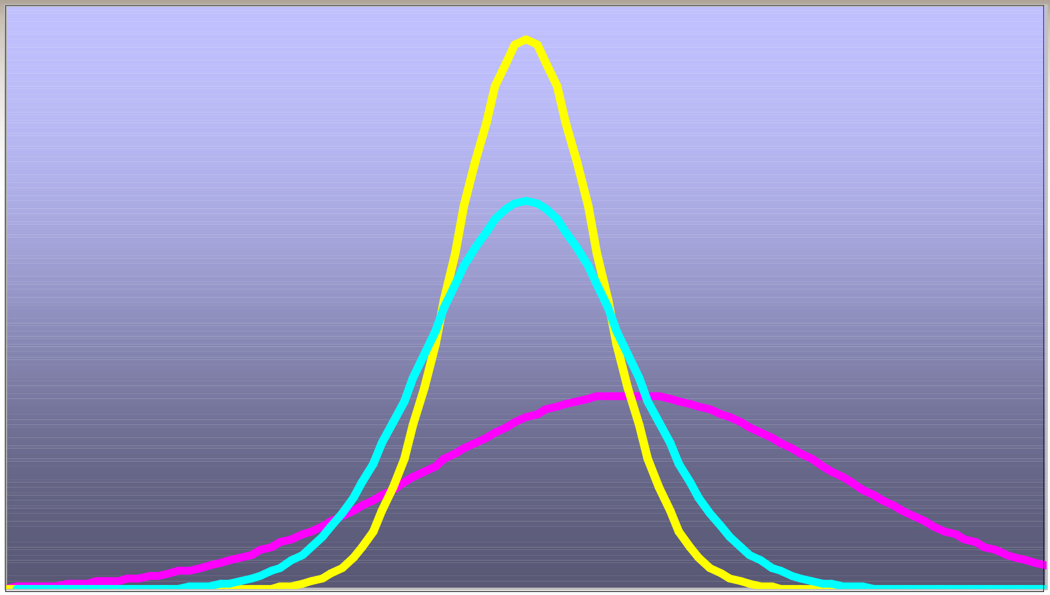
Mean is 0

Standard Deviation is 1



Values above the mean have positive Z-values, Values below the mean have negative Z-values

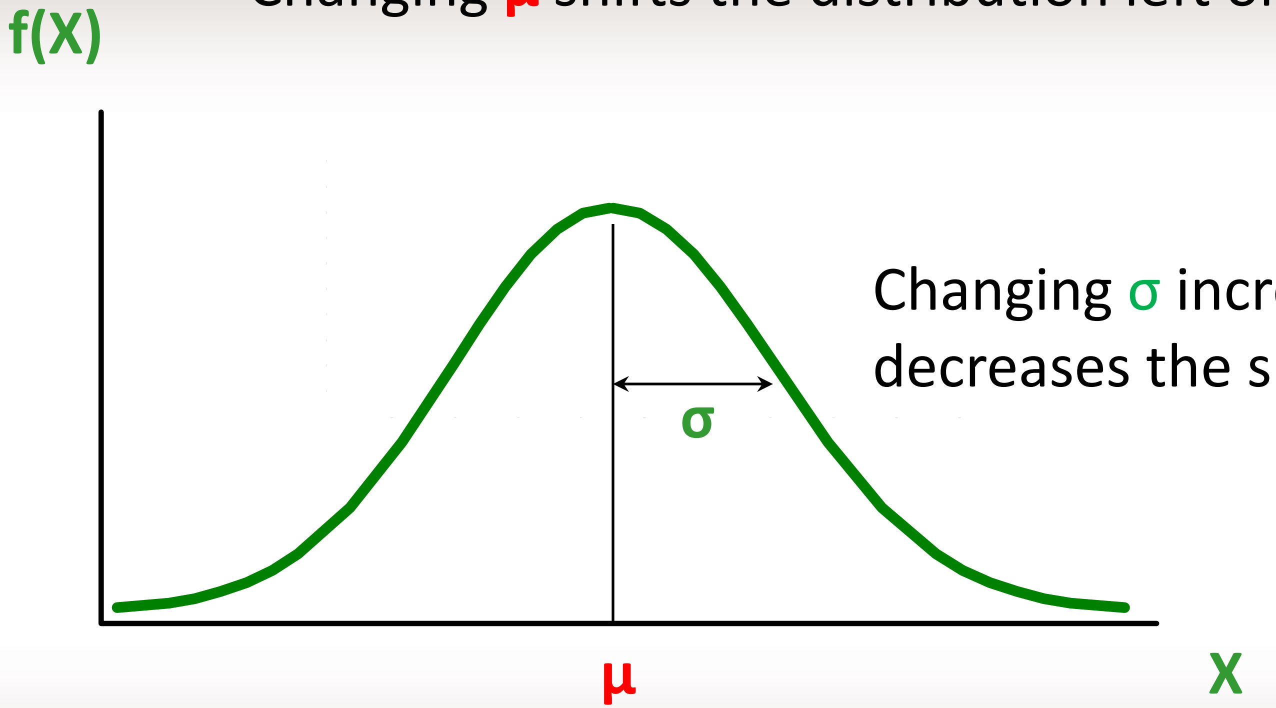
Many Normal Distributions



By varying the parameters μ and σ, we obtain different normal distributions

The Normal Distribution Shape

Changing μ shifts the distribution left or right.



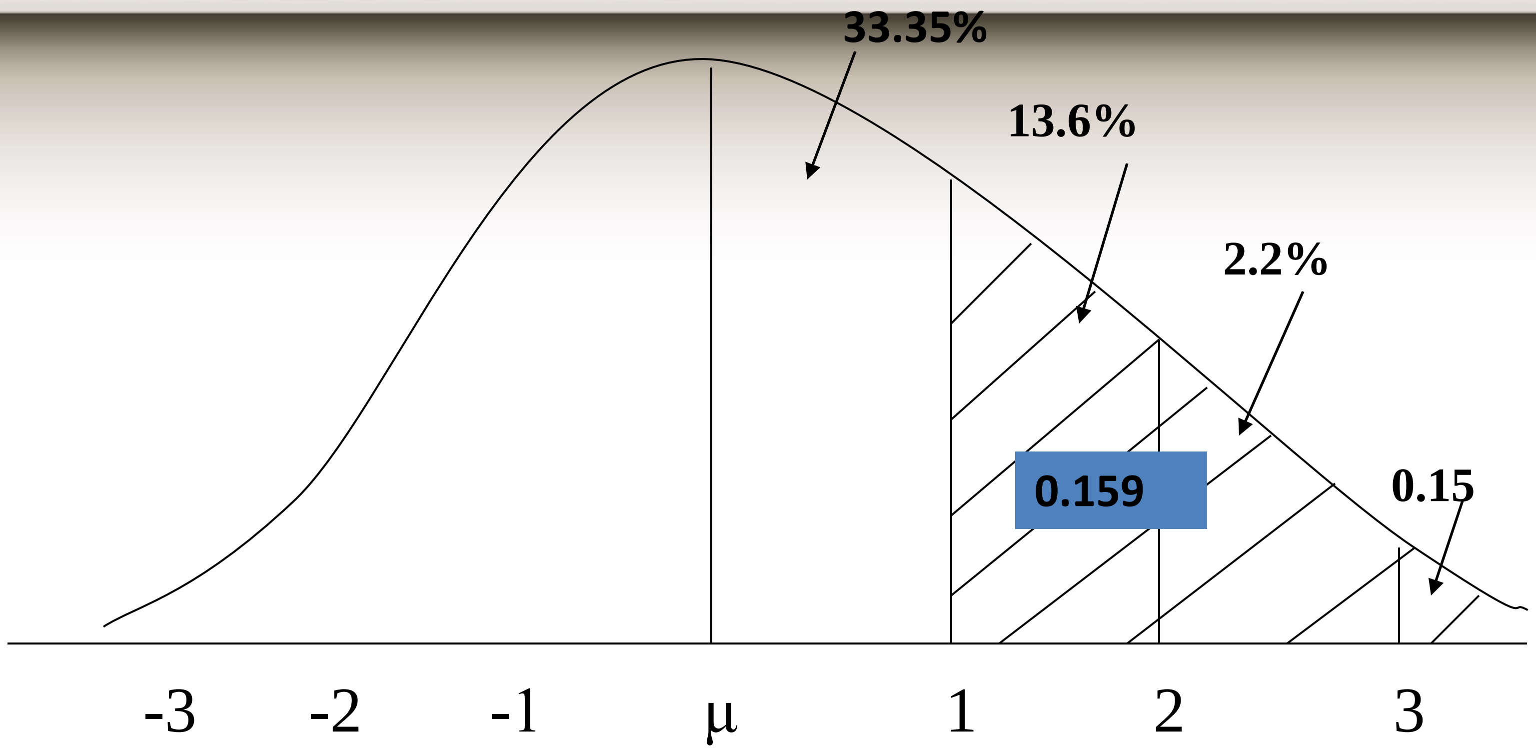
Changing σ increases or decreases the spread μ

Exercise # 1

Then:

1) What area under the curve is above 80 beats/min?

Diagram of Exercise # 1

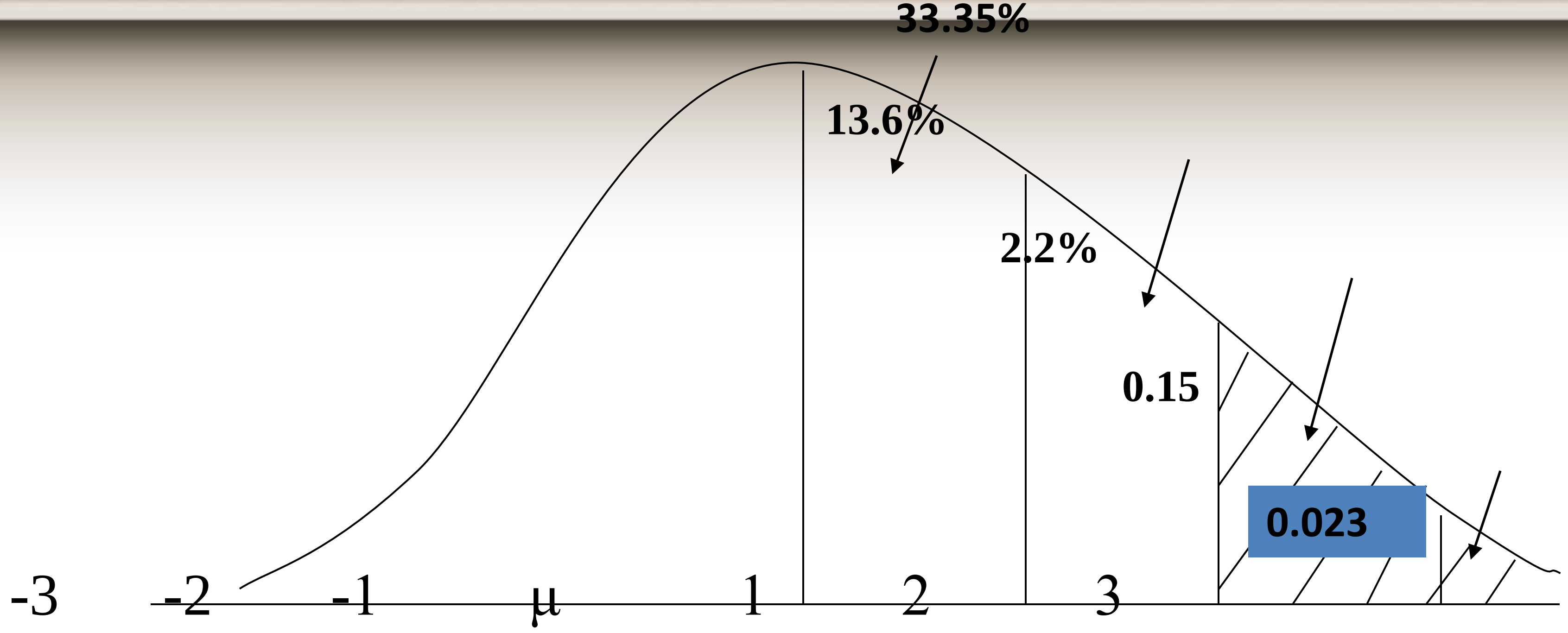


Exercise # 2

Then:

2) What area of the curve is above 90 beats/min?

Diagram of Exercise # 2

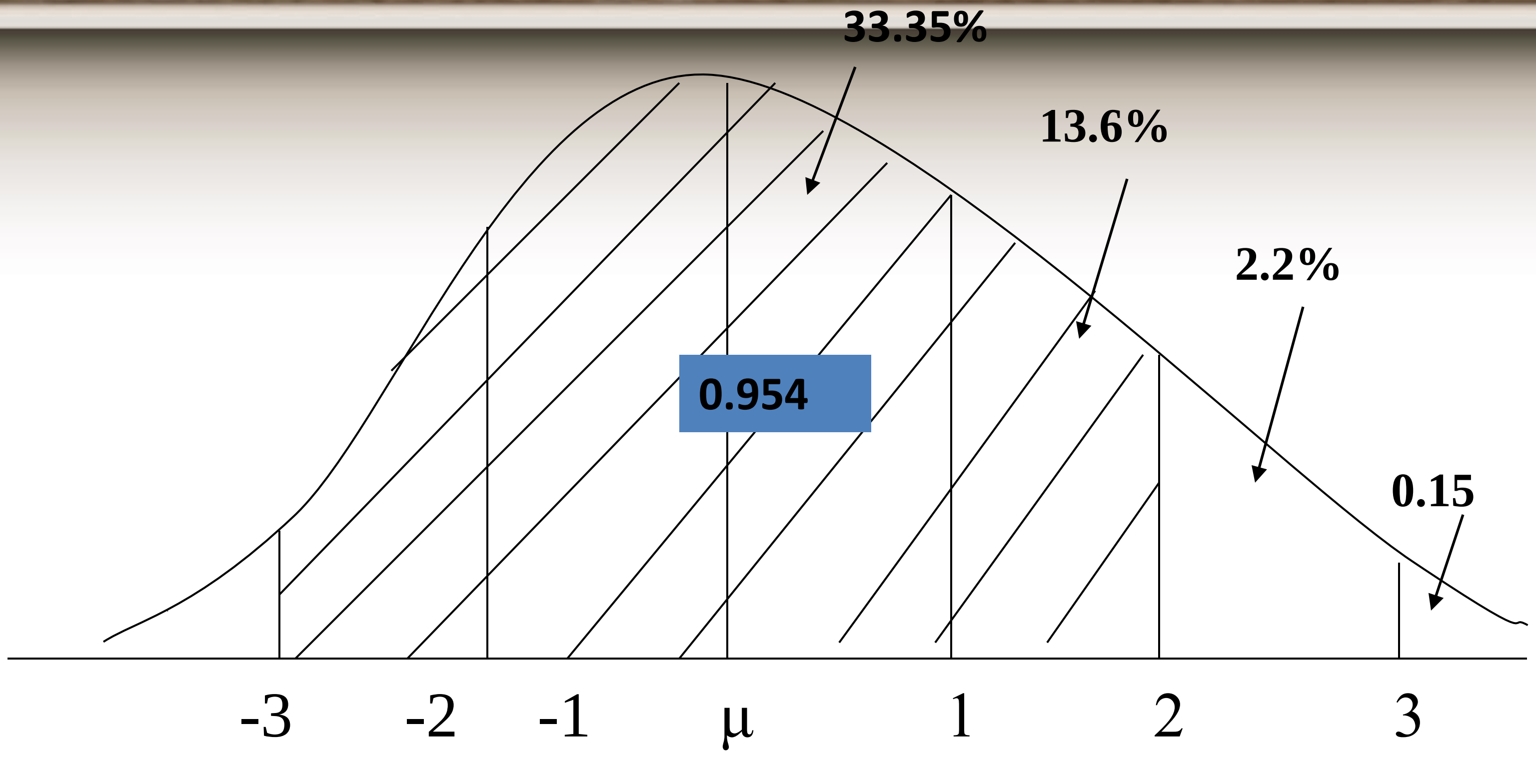


Exercise # 3

Then:

3) What area of the curve is between 50-90 beats/min?

Diagram of Exercise # 3



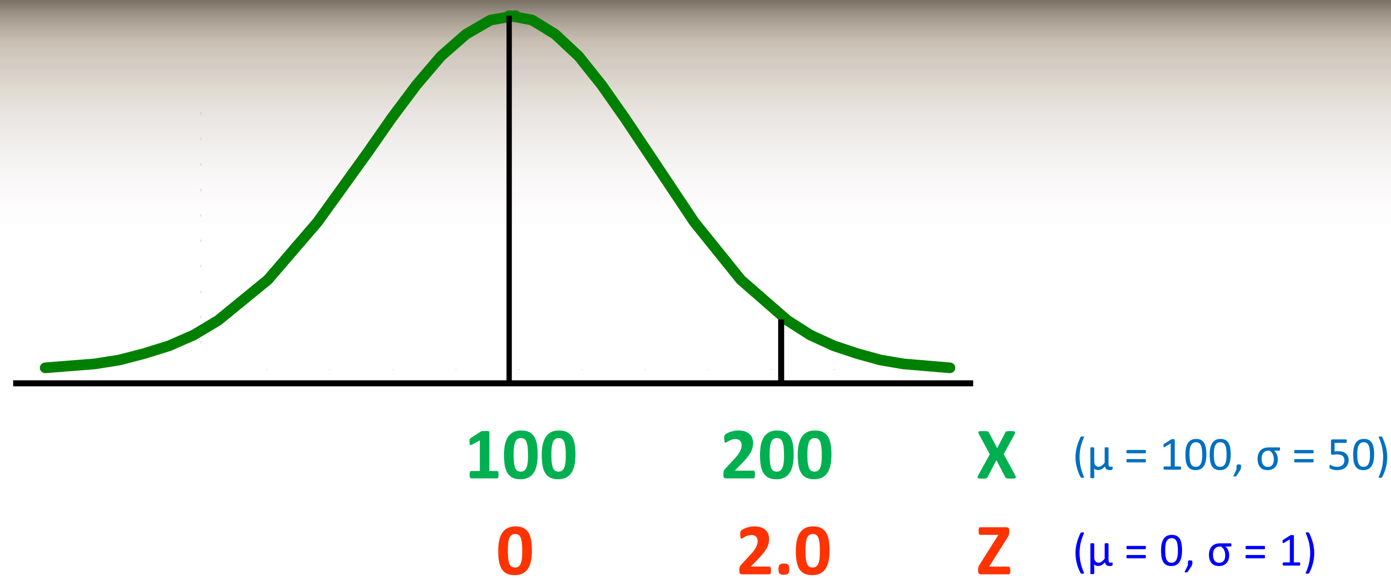
Example

If X is distributed normally with mean of 100 and standard deviation of 50, the Z value for X = 200 is

$$Z = \frac{X - \mu}{\sigma} = \frac{200 - 100}{50} = 2.0$$

This says that **X = 200** is two standard deviations (2 increments of 50 units) above the mean of 100.

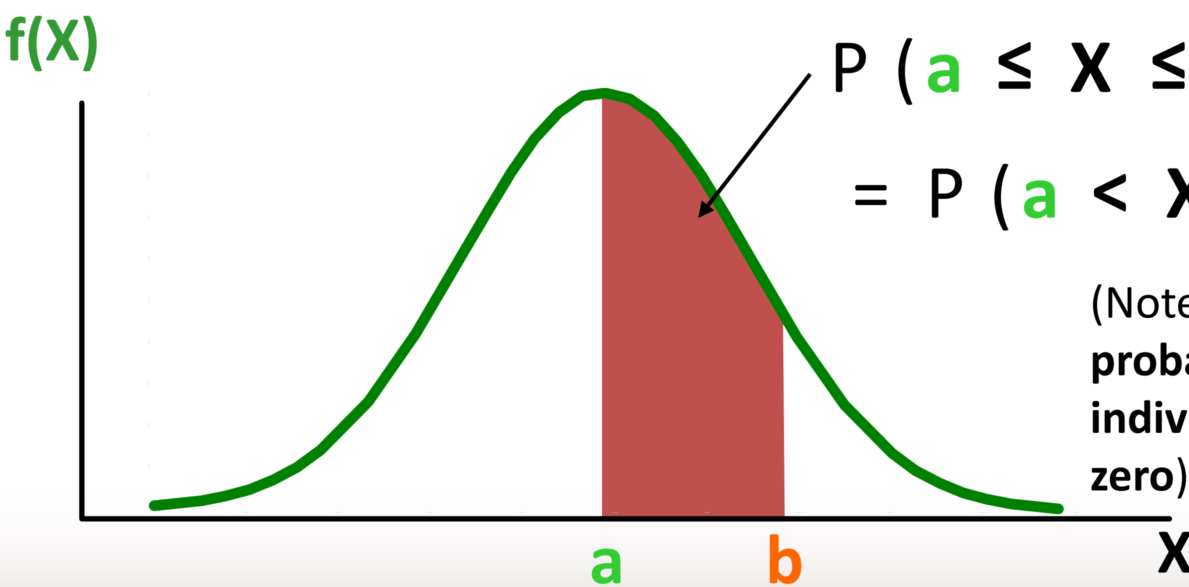
Comparing X and Z units



Note that the shape of the distribution is the same, only the scale has changed. We can express the problem in original units (X) or in standardized units (Z)

Finding Normal Probabilities

Probability is measured by the area under the curve



$$P(a \le X \le b)$$

$$= P(a < X < b)$$

(Note that **the probability of any individual value is zero**)

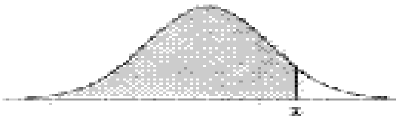
Notation for the Probability of a Standard Normal Random Variable

$P(a < Z < b)$ represents the probability a standard normal random variable is between *a* and *b*

$P(Z > a)$ represents the probability a standard normal random variable is greater than *a*.

$P(Z < a)$ represents the probability a standard normal random variable is less than *a*.

Tables of the Normal Distribution



Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

The Standardized Normal Table

(continued)

The column gives the value of Z to the second decimal point

Z	0.00	0.01	0.02 …
0.0			
0.1			
.			
.			
.			
2.0	.9772		

The row shows the value of Z to the first decimal point

The value within the table gives the probability from Z = – ∞ up to the desired Z value

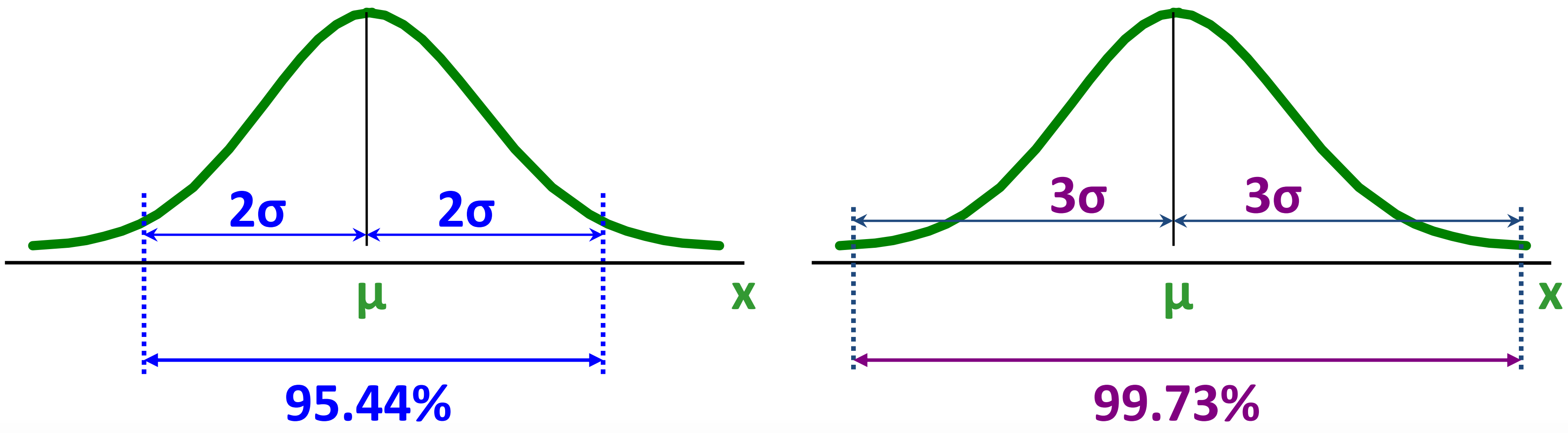
P(Z < 2.00) = 0.9772

The Empirical Rule

(continued)

μ ± 2σ covers about 95% of X's

μ ± 3σ covers about 99.7% of X's



Comparison between binomial and normal distributions

	Binomial	Normal
variable is represented by:	r (r counts the number of successes)	x (x is a measurement variable)
type of variable	discrete (takes on only whole number values)	continuous (takes on every value in an interval)
graph	Histogram with n+1 bars; one each for r=0, r=1, ..., r=n	bell-shaped curve
To answer probability question concerning the probability of the variable being in a specified interval	Add the areas within appropriate bars	convert to z-scores and use standard normal probability table for areas