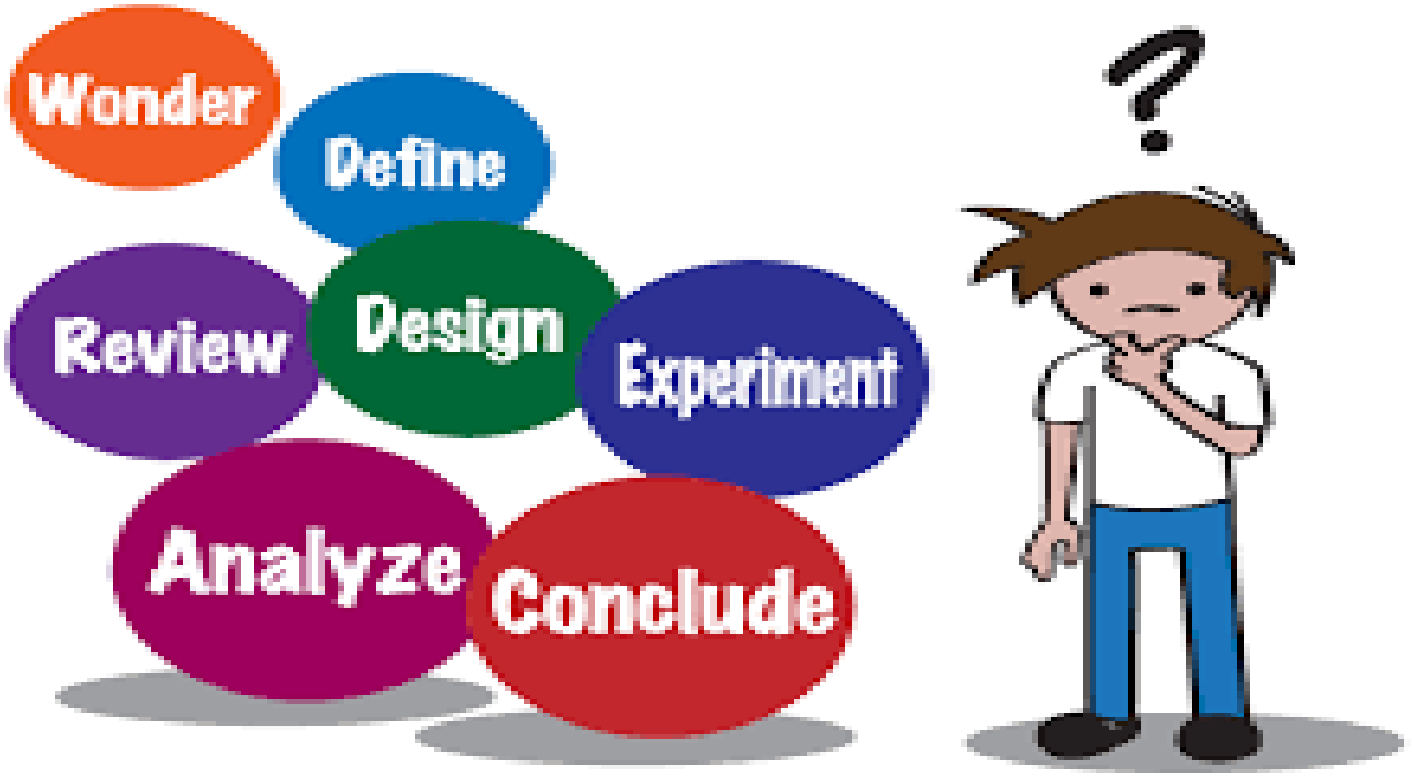
HYPOTHESIS TESTING



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OUTLINES

- Introduction
- Types of statistics
- Hypothesis
- Testing of hypothesis
- Characteristics
- Role
- Types of hypothesis
- Errors in hypothesis test
- Steps
- Type of test

Terms Introduce

- **Population** ≡ all possible values
- **Sample** ≡ a portion of the population
- **Statistical inference** ≡ generalizing from a sample to a population with calculated degree of certainty

  Two forms of statistical inference
  - **Hypothesis testing**
  - **Estimation**
- **Parameter** ≡ a characteristic of population, e.g., population mean μ
- **Statistic** ≡ calculated from data in the sample, e.g., sample mean ($\bar{x}$)

Types of Statistics

- Descriptive statistics:
  - Organize and summarize scores *from samples*
- Inferential statistics:
  - Infer information *about the population* based on what we know from sample data
  - Decide if an experimental manipulation has had an effect

THE BIG PICTURE OF STATISTICS

Theory

↓

Question to answer / **Hypothesis** to test

↓

Design Research Study

↓

Collect **Data**
(measurements, observations)

↓

Organize and make sense of the #s
USING STATISTICS!
Depends on our goal:

Describe characteristics

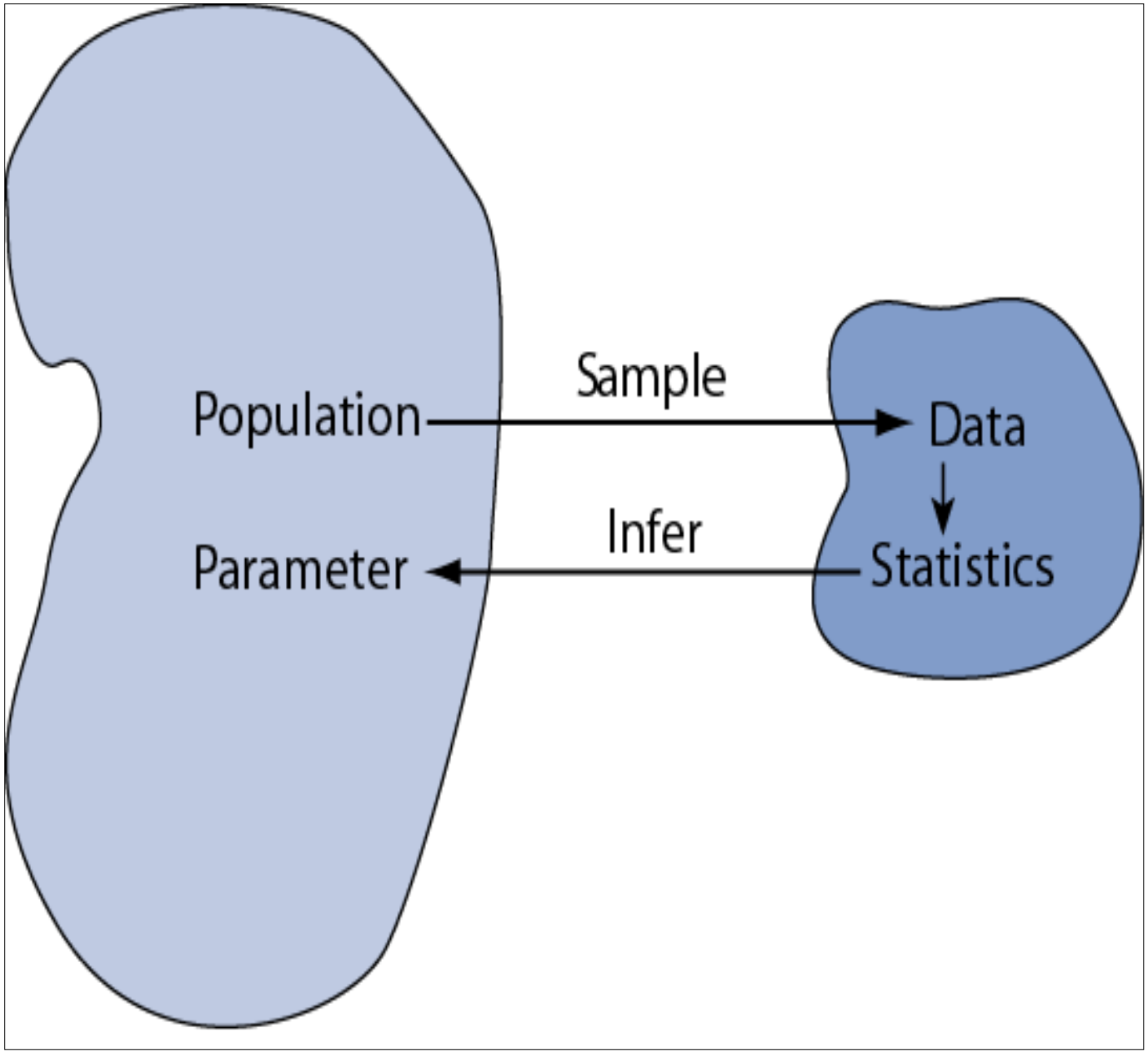
organize, summarize, condense data

DESCRIPTIVE STATISTICS

Test hypothesis, Make conclusions,

interpret data, understand relations

INFERENTIAL STATISTICS

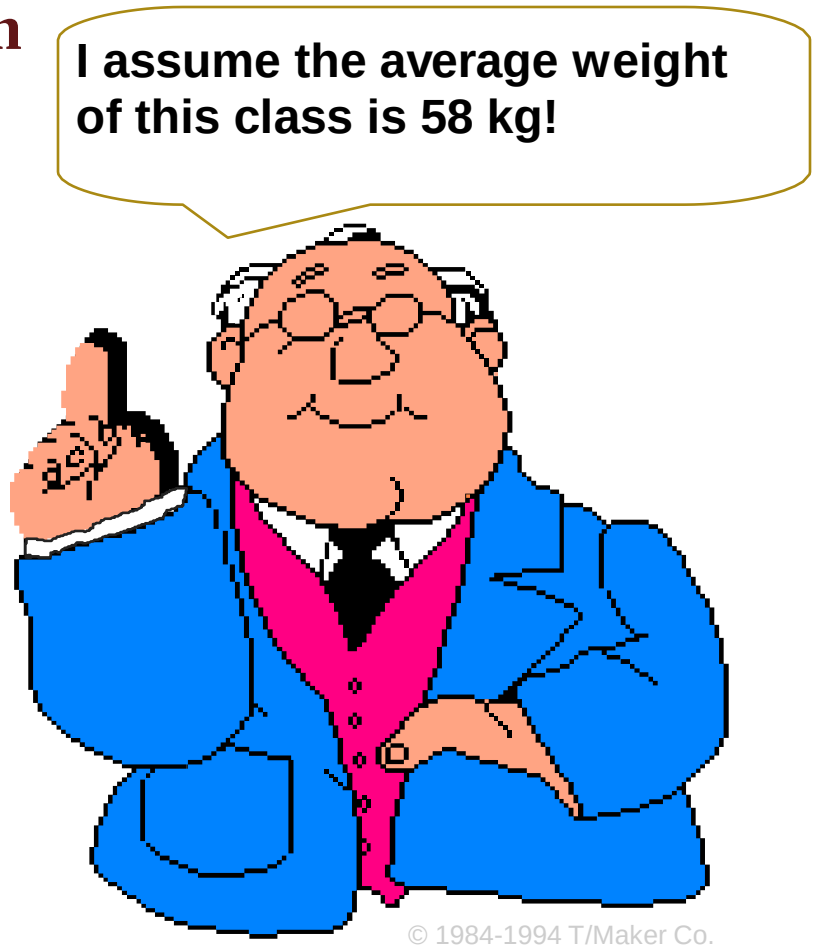

Population
Sample
Data
Infer
Parameter
Statistics

What is a Hypothesis?

A hypothesis is an assumption about the population parameter.

A parameter is characteristic of the population, like its mean or variance.

The parameter must be identified before analysis.



THE PURPOSE OF A HYPOTHESIS

A hypothesis should always:

- explain what you expect to happen
- be clear and understandable
- be testible
- be measurable
- contain an independent and dependent variable

HYPOTHESIS

- It is a statement about the population parameter.
- In other words, it is a conclusion which is tentatively drawn on logical basis.

HYPOTHESIS TESTING

- Initiated by J Neyman & E S Pearson. It employs statistical techniques to arrive at decision in certain situations where there is an element of uncertainty on the basis of sample, whose size is fixed in advance.

Testing of Hypothesis

A hypothesis is an assumption about the population parameter (say population mean) which is to be tested.

For that we collect sample data , then we calculate sample statistics (say sample mean) and then use this information to judge/decide whether hypothesized value of population parameter is correct or not.

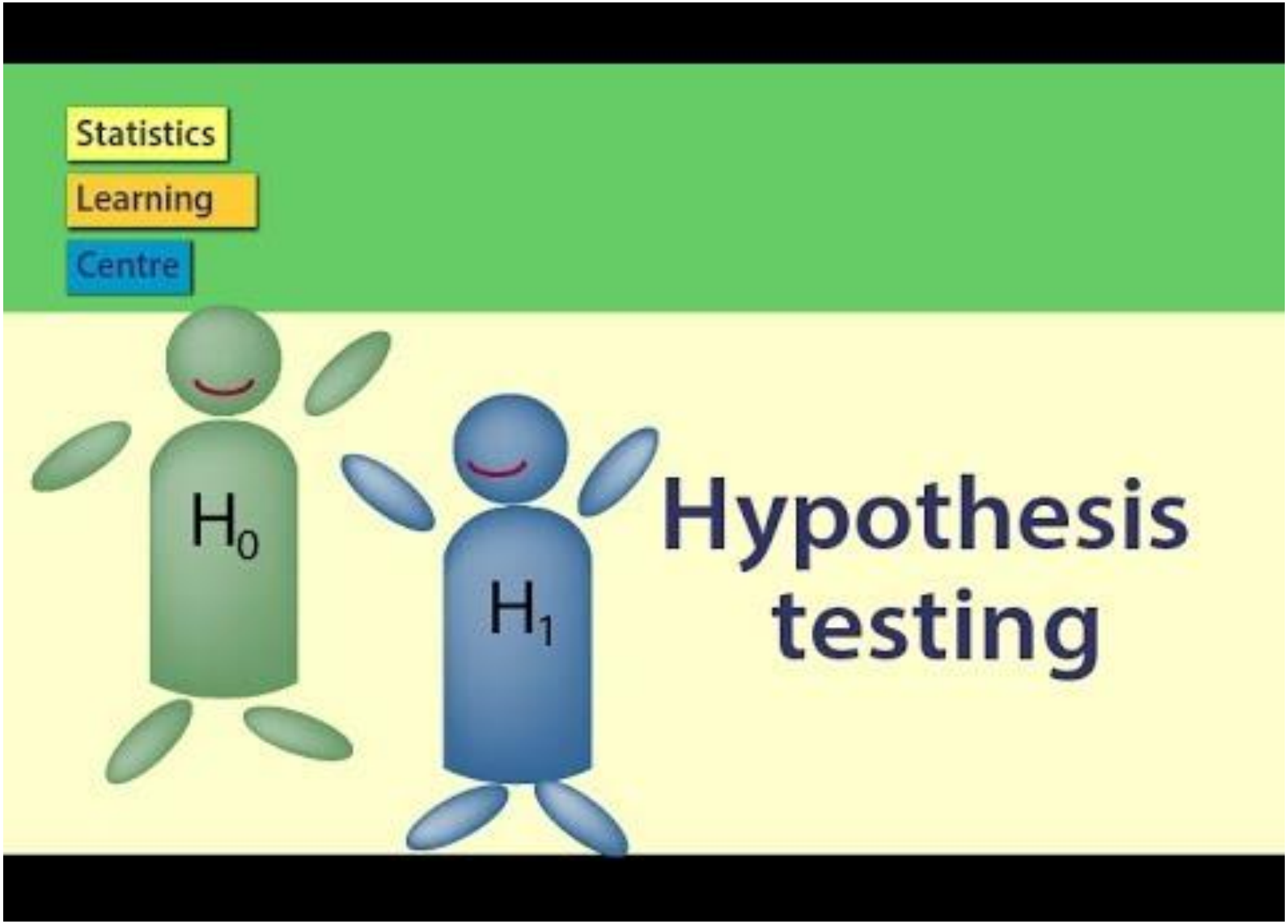
Characteristics of Hypothesis

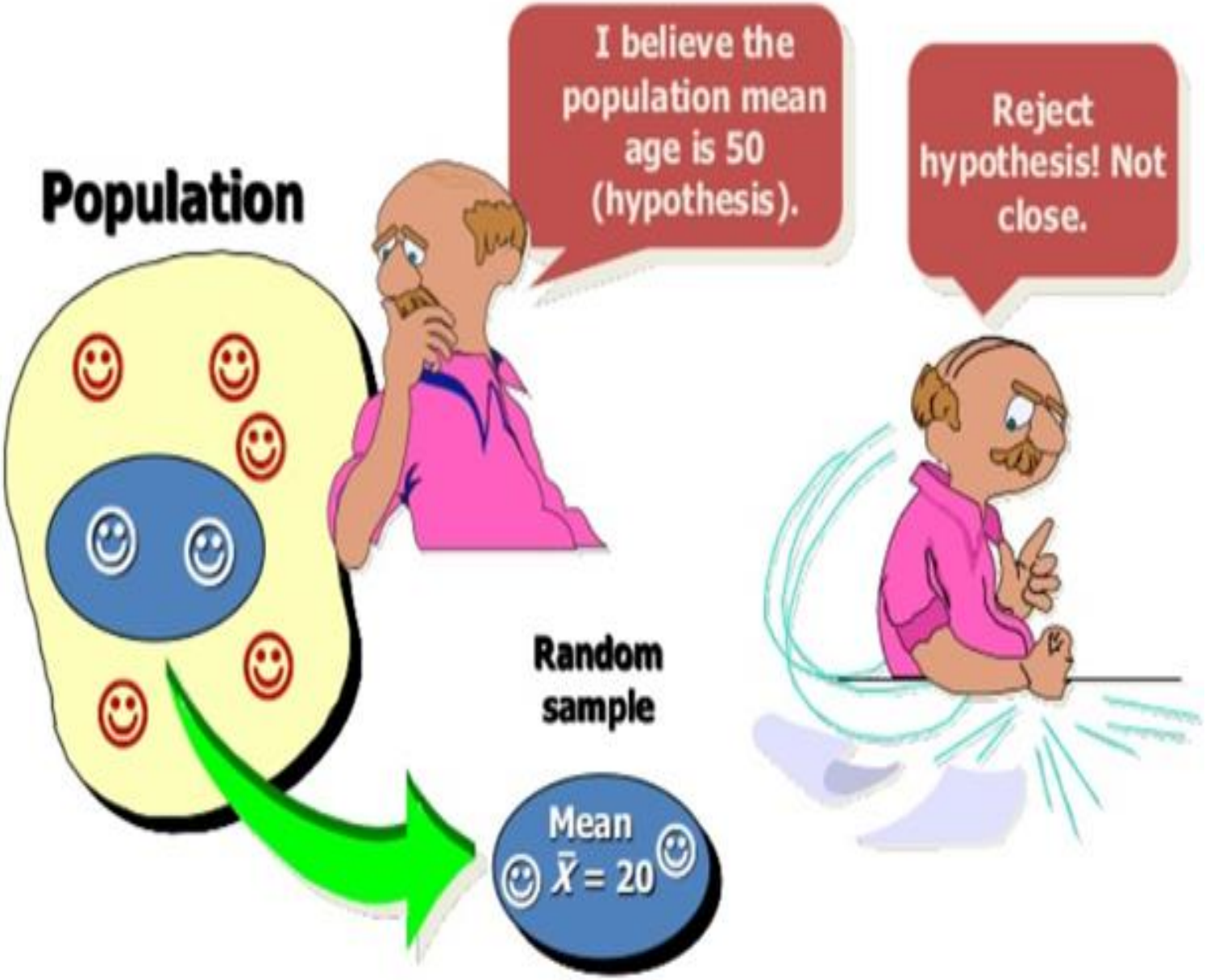
- It should be clear and precise
- Should be capable of being tested
- It should state the relationship between variables
- It should be limited by scope and be specific
- It should be stated as far as possible with most simple terms so that the same is easily understandable by all concerned
- It should be consisted with most known facts
- It should be amenable to testing within a reasonable time
- Must explain the facts that gave rise to the need for explanation

Role

- Helps in guide the investigator in the right decision
- It is clear to the researcher through hypothesis about the study
- Type of research – exploratory, descriptive or causal is decided by hypothesis
- Statistical techniques is determined
- It formulated & tested, if it is found to be true becomes a part of accepted theory.

Types of hypothesis




Population
I believe the population mean age is 50 (hypothesis).
Reject hypothesis! Not close.
Random sample
Mean
$\bar{X} = 20$

HYPOTHESIS TESTING

Null hypothesis, H_0

- State the hypothesized value of the parameter before sampling.

The assumption we wish to test or the assumption we are trying to reject.

- E.g population mean $\mu = 20$
- There is no difference between coke and diet coke

Alternative hypothesis, H_A

All possible alternatives other than

E.g $\mu \neq 20$

$\mu > 20$

$\mu < 20$

There is a difference between coke

NULL HYPOTHESIS

The null hypothesis H_0 represents a theory that has been put forward either because it is believed to be true or because it is used as a basis for an argument and has not been proven.

- For example, in a clinical trial of a new drug, the null hypothesis might be that the new drug is no better, on average, than the current drug. We would write

- H_0: there is no difference between the two drugs on an average.


TYPE I ERRORS
null hypothesis:
dogs live longer
than cats
dogs don't live
longer than cats

ALTERNATIVE HYPOTHESIS

The alternative hypothesis, H_A, is a statement of what a statistical hypothesis test is set up to establish.

For example, in the clinical trial of a new drug, the alternative hypothesis might be that the new drug has a different effect, on average, compared to that of the current drug. We would write

H_A: the two drugs have different effects, on average.

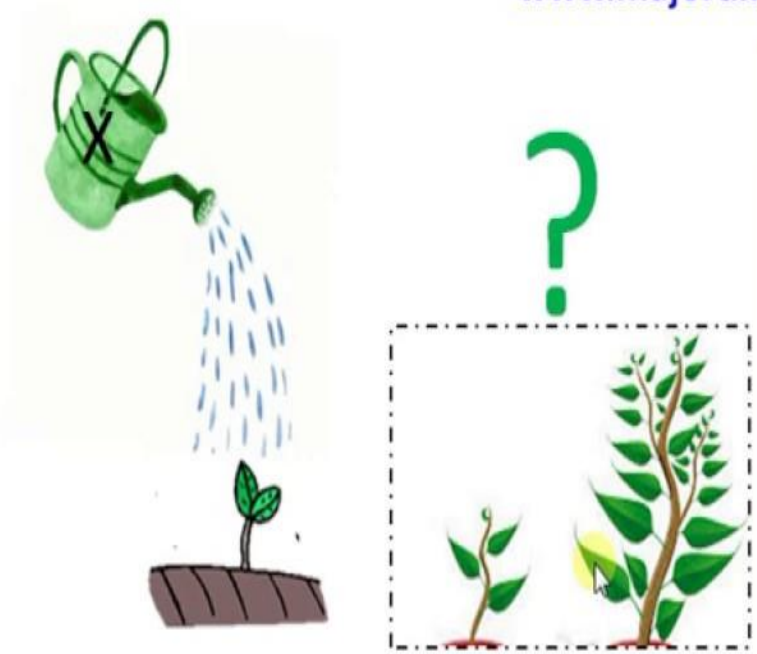
Or

H_A: the new drug is better than the current drug, on average.

The result of a hypothesis test:

'Reject H_0 in favour of H_A OR 'Do not reject H_0

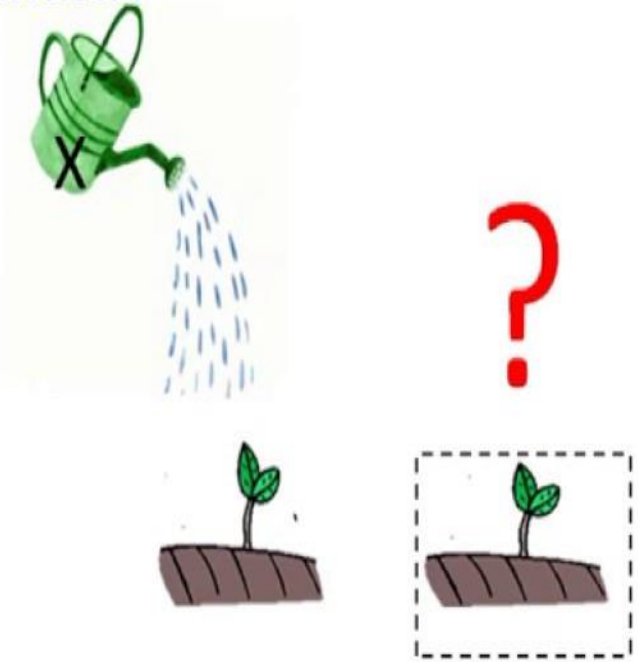
www.majordifferences.com



H_1: Application of bio-fertilizer 'x' increase plant growth.

Alternative hypothesis

✓The alternative hypothesis is a hypothesis which the researcher tries to prove.



H_0: Application of bio-fertilizer 'x' do not increase plant growth.

Null hypothesis

✓The null hypothesis is a hypothesis which the researcher tries to disprove, or nullify.

The Null Hypothesis, H_0

- State the Assumption (numerical) to be tested
- e.g. The average weight of the semester 1 student is 58kgs (H_0: 58)58)
- Begin with the assumption that the null hypothesis is TRUE.

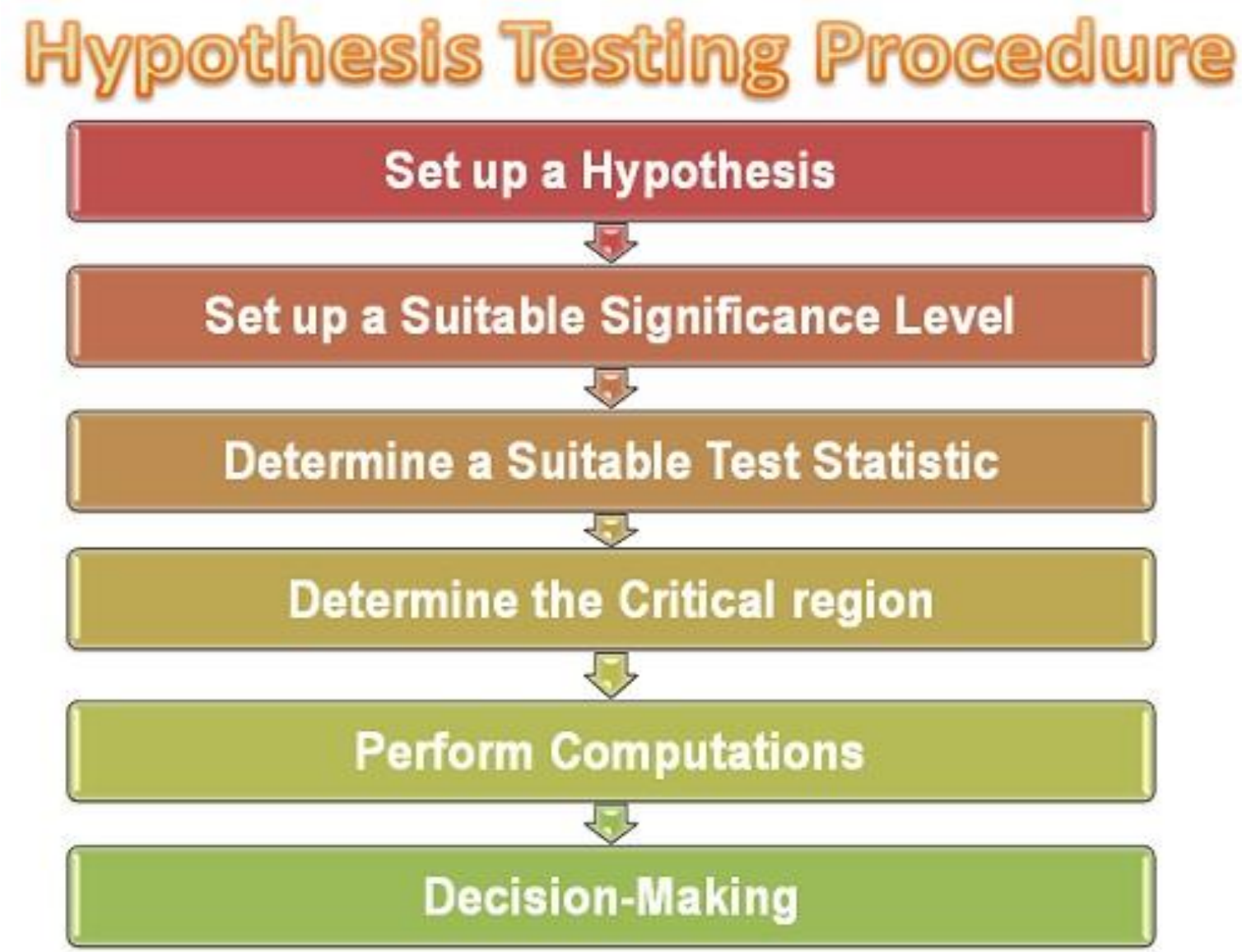
(Similar to the notion of innocent until proven guilty)

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis

eg. The average weight of the students is not equal to 58kgs. (H_1: $\mu \neq 58$)

Steps


Hypothesis Testing Procedure
Set up a Hypothesis
Set up a Suitable Significance Level
Determine a Suitable Test Statistic
Determine the Critical region
Perform Computations
Decision-Making

Procedure of Hypothesis Testing

The Hypothesis Testing comprises the following steps:

Step 1

Set up a hypothesis.

Step 2

Set up a suitable significance level.

The confidence with which an experimenter rejects or accepts Null Hypothesis depends on the significance level adopted**.** Level of significance is the **rejection region (** which is outside the confidence

or acceptance region).The level of significance, usually denoted by the

α.

Selecting a significance level

Though any level of significance can be adopted, **in practice we either take 5% or 1% level of significance .**

When we take 5% level of significance($\alpha = .05$), then there are about 5 chances out of 100 that we would reject the null hypothesis. In other words out of 100, 95% chances are there that the null hypothesis will be accepted i.e. we are about 95% confident that we have made the right decision.

p-value

- The probability, assuming the null hypothesis is true, of observing a result at least as extreme as the test statistic.

Degree of freedom

- It tells the researcher the number of elements that can be chosen freely.

When p value is less than calculated value, null hypothesis is accepted.

When p value is more than calculated value , null hypothesis is rejected

If our sample statistic(calculated value) fall in the non-shaded region(acceptance region), then it simply means that there is no evidence to reject the ***null hypothesis.***

It proves that null hypothesis (H_0) is true. Otherwise, it will be rejected.

Step 3

Determination of suitable test statistic: For example Z, t Chi-Square or F-statistic.

Statistical test

A procedure whose inputs are samples and whose result is a hypothesis.

Region of acceptance

- The set of values of the test statistic for which we fail to reject the null hypothesis.

Region of rejection / Critical region

- The set of values of the test statistic for which the null hypothesis is rejected.

Step 4

Determine the critical value from the table.

Critical value

- The threshold value delimiting the regions of acceptance and rejection for the test statistic.

Step 5

After doing computation, check the sample result.

Compare the calculated value(sample result) with the value obtained from the table.(tabulated or critical value)

Step 6

Making Decisions

Making decisions means either accepting or rejecting the null hypothesis.

If computed value(absolute value) is more than the tabulated or critical value, then it falls in the critical region. In that case, reject null hypothesis, otherwise accept.

Type I and Type II Errors

When a statistical hypothesis is tested, there are 4 possible results:

(1) The hypothesis is true but our test accepts it.

(2) The hypothesis is false but our test rejects it.

(3) The hypothesis is true but our test rejects it.

(4) The hypothesis is false but our test accepts it.

Obviously, the last 2 possibilities lead to errors.

Rejecting a null hypothesis when it is true is called a ***Type I error.***

Accepting a null hypothesis when it is false is called ***Type II error.***

	DATA ANALYSIS OUTCOME	
In Population	***Accept Null Hypothesis***	***Reject Null Hypothesis***
Null Hypothesis True	Correct Decision	Type I Error
Null Hypothesis False	Type II Error	Correct Decision

Example 1 - Court Room Trial

In court room, a defendant is considered not guilty as long as his guilt is not proven. The prosecutor tries to prove the guilt of the defendant. Only when there is enough charging evidence the defendant is condemned.

In the start of the procedure, there are two hypotheses

H_0: "the defendant is not guilty", and

H_1: "the defendant is guilty".

The first one is called ***null hypothesis***, and the second one is called ***alternative hypothesis***.

	Null Hypothesis (H_0) is true He is not guilty	Alternative Hypothesis (H_1) is true He is guilty
Accept Null Hypothesis	Right decision	Wrong decision Type II Error
Reject Null Hypothesis	Wrong decision Type I Error	Right decision

One-Tailed and Two-Tailed Tests

Two-Tailed Test is that where the hypothesis about the population parameter is rejected for the value of sample statistic failing into **either tail** of the distribution.(fig3)

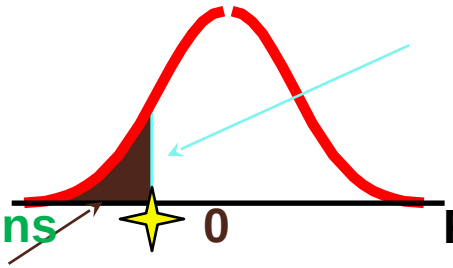
When the hypothesis about the population parameter is rejected for the value of sample statistic failing into **one side tail** of the distribution, then it is known as **one-tailed test.**

If the rejection area falls on the right side, then it is called right-tailed test.(fig 2) On the other hand If the rejection area falls on the right side, then it is called left-tailed test.(fig 1)

Two-tail and One-tail Test

H_0: 3

H_1: < 3



Critical Value(s)

Rejection Regions

Fig 1 One-tail Test

H_0: 3

H_1: > 3

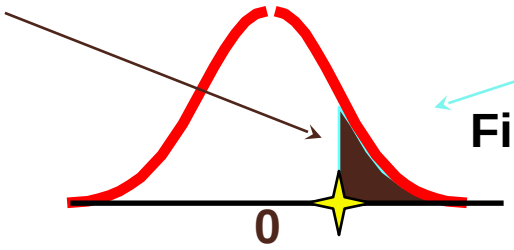


Fig 2 One-tail Test

H_0: 3

H_1: 3

/2

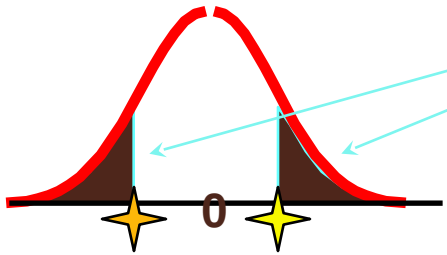


Fig-3 two-tail test

Summary of One- and Two-Tail Tests…

One-Tail Test (left tail)	Two-Tail Test	One-Tail Test (right tail)
$H_0 : u = \sim u_0$	$Ho{:}\mu = \mu o$	$Ho{:}\mu = \mu o$
$H1 :: u < \mu o$	$H_1 : {}_1u \sim !lo$	$H_1 : \mu > \mu_0$

The following table gives critical values of Z for both one-tailed and two- tailed tests at various levels of significance.

level of significance	0.10	.05	.01
critical value of z for one-tailed test	1.28 or -1.28	1.645 or -1.645	2.33 0r -2.33
critical value of z for two-tailed test	1.645 or -1.645	1.96 or -1.96	2.58 or -2.58

Types of test

Parametric Tests

- Parametric Tests or Standard tests of Hypothesis.
- They are more powerful. The data in the test is derived from interval & ration measurement. Is based on assumption on certain charactertics.

The important parametric tests are:-

- Z- Test
- T-Test
- X- Test and
- F- Test

Non –Parametric tests

Non –Parametric tests or Distribution free test of Hypothesis

It not make any assumption. These are distribution free test.

A. One sample and Two sample tests

- Binomial test
- Chi- square test
- McNemar test

B. K- sample tests (K > 3)

- Kruskal-Wallis test : H
- Friedman test
- KKendall`s coefficient of concordance: W


NULL HYPOTHESIS
Means no relationship between two variables.
ALTERNATIVE HYPOTHESIS
Means rejection of null hypothesis.
Buzzle.com

example


A Statistician Responds to a Marriage Proposal
I Reject the Null Hypothesis.
Will you marry me?
Huh? What does that mean?
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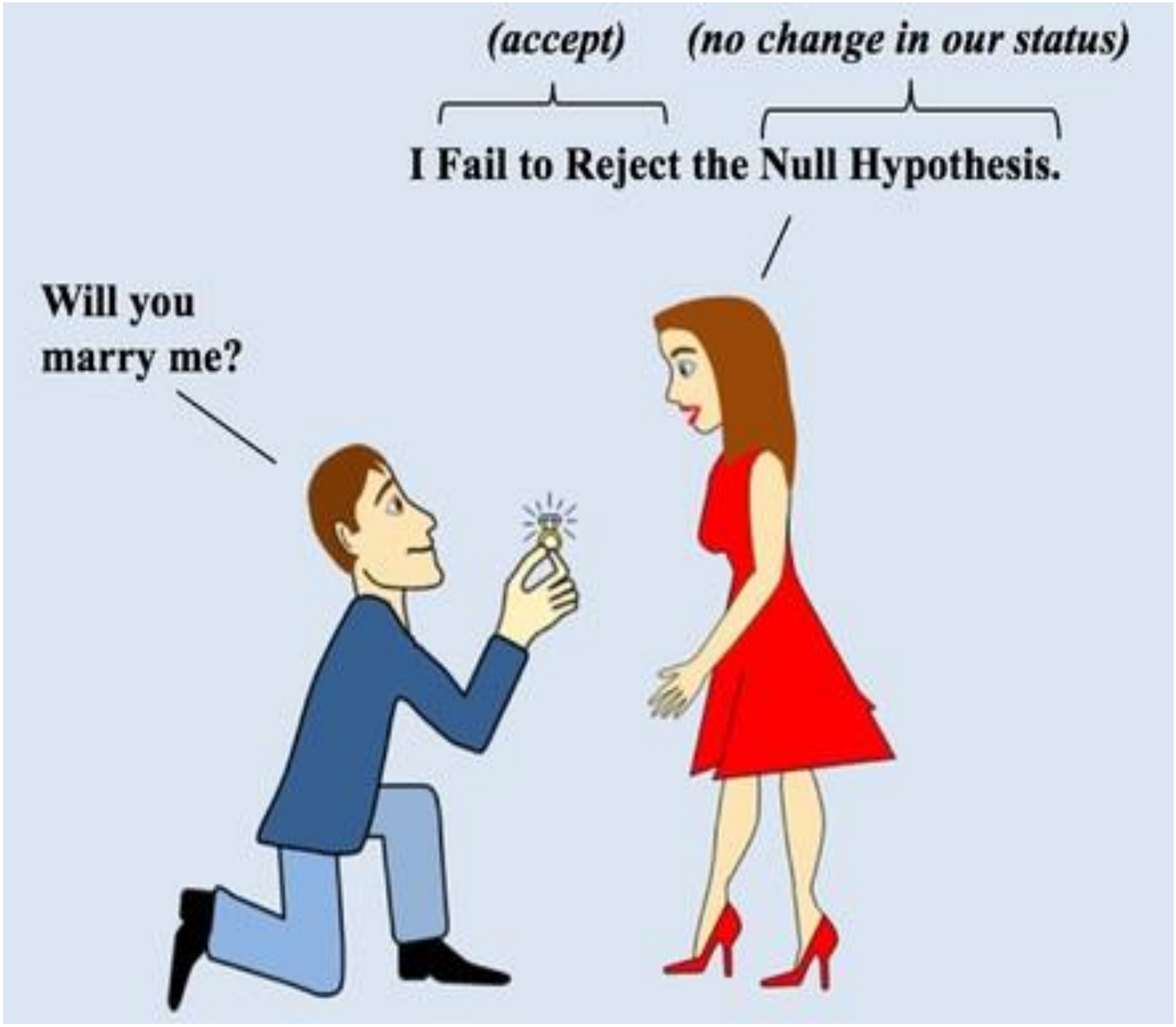
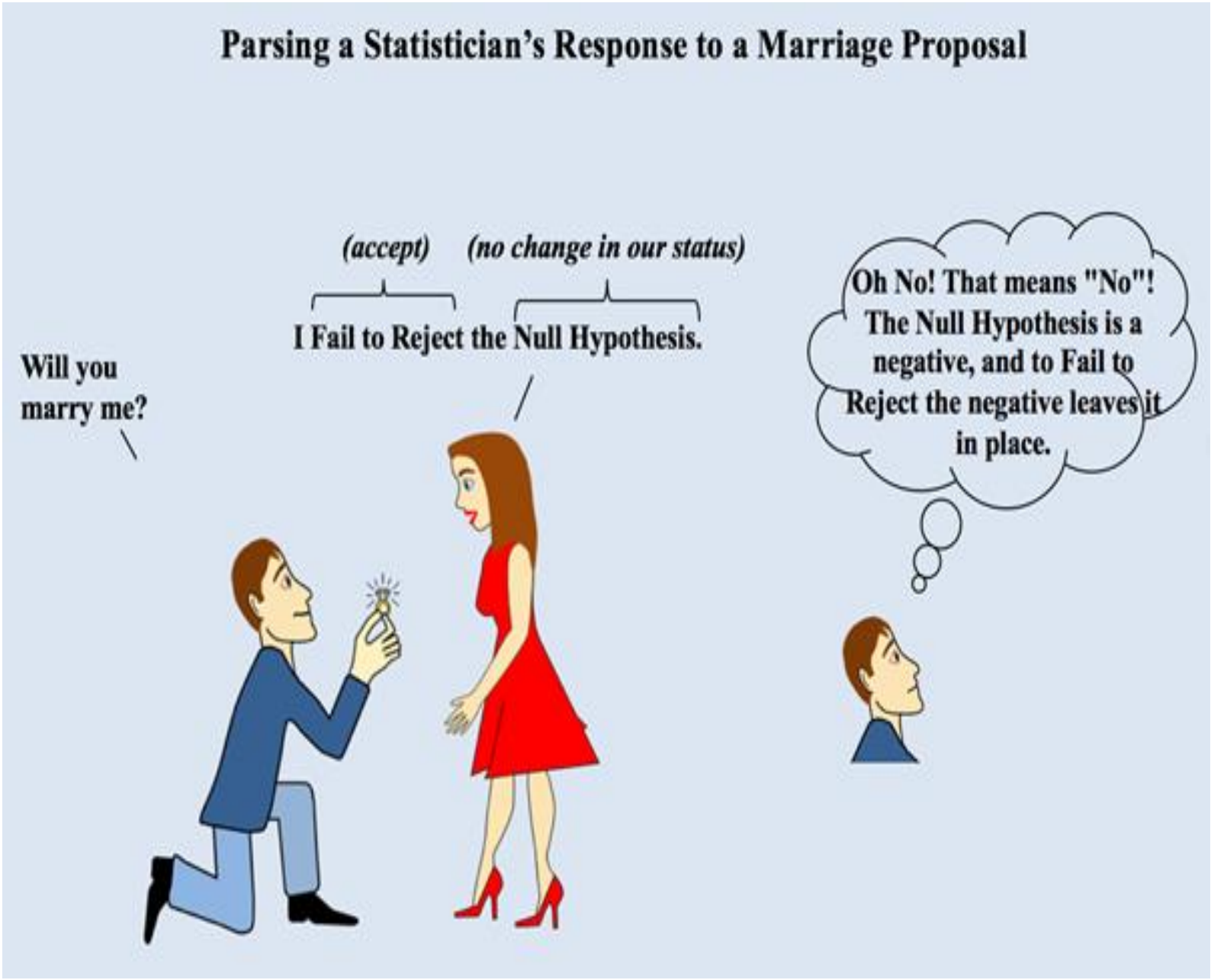
from the book Statistics from A to Z — Confusing Concepts Clarified.


(accept)
(no change in our status)
I Fail to Reject the Null Hypothesis.
Will you marry me?


Parsing a Statistician's Response to a Marriage Proposal
(accept)
(no change in our status)
I Fail to Reject the Null Hypothesis.
Will you marry me?
Oh No! That means "No"! The Null Hypothesis is a negative, and to Fail to Reject the negative leaves it in place.

Null Hypothesis

Timmy Brushed His Teeth

Alternative Hypothesis

Timmy Did Not Brush His Teeth.

The Data

A Dry Toothbrush

A Rejection of the Null

5 Minutes Earlier